Introduction

West Virginia's College- and Career-Readiness Standards have been developed with the goal of preparing students to be successful in a wide range of high-quality post-secondary opportunities. Specifically, college- and career-readiness refers to the knowledge, skills, and dispositions needed to be successful in higher education and/or training that lead to gainful employment. The West Virginia College- and Career-Readiness Standards establish a set of knowledge and skills that all individuals need to transition into higher education or the workplace, as both realms share many expectations. Throughout their educational experience students should develop a full understanding of the career opportunities available, the education necessary to be successful in their chosen pathway, and a plan to attain their goals.

West Virginia's College- and Career-Readiness Standards for Mathematics (WVCCRSM) are the culmination of an extended, broad-based effort to help ensure that all students are college- and career-ready upon completion of high school. The skills contained in the mathematics standards are essential for college- and career-readiness in a twenty-first-century, globally competitive society. The standards reflect a progression and key ideas determining how knowledge is organized and generated within the content area. Standards evolve from specifics to deeper structures inherent in the discipline. These deeper structures serve to connect the specifics. The standards follow such a design, stressing conceptual understanding of key ideas and continually returning to organizing principles such as place value or the properties of operations to structure those ideas. The sequence of topics and performances outlined in mathematics standards must respect the scientific research about how students learn and what is known about how their mathematical knowledge, skill, and understanding develop over time.

The West Virginia College- and Career-Readiness Standards for Mathematics (WVCCRSM) are the result of an intensive review of the current mathematics standards adopted by the West Virginia Board of Education and placed into policy on July 1, 2016. Teachers and representatives from institutions of higher education from around the state met during the summer and fall of 2022 to discuss the mathematics standards and determined how to strengthen the progression of learning from grade level to grade level and course to course. While there are several changes within grade levels and courses, the changes are derived from the most current academic and scientific research and the lessons learned during the implementation of the 2016 version of the standards. Additionally, in January of 2023, the WVCCRSM was evaluated by a diverse team of stakeholders who made recommendations to the West Virginia Department of Education based on the needs of West Virginia students and college/career requirements. The suggestions and recommendations are incorporated into the 2024 WVCCRSM and reflect the improvements determined during the review process.

Explanation of Terms

Clusters are groups of standards that define the expectations students must demonstrate to be college- and career-ready.

Domains are broad components that make up a content area. Domains vary by grade-level and by course. For example, the five domains for mathematics in Grade 6 are Ratios and Proportional Reasoning, The Number System, Expressions and Equations, Geometry, and Statistics and Probability.

Standards are expectations for what students should know, understand, and be able to do; standards represent educational goals.

Number of Standards

The number for each standard is composed of three parts, each part separated by a period:

a. the content code (e.g., M for Mathematics),
b. the grade level or course, and
c. the standard.

Illustration: M.3.1 refers to mathematics, grade 3, standard 1; M.C.1 refers to mathematics, calculus, standard 1.

Mathematics Grade Level or Course:

K	Kindergarten
1	Grade 1
2	Grade 2
3	Grade 3
4	Grade 4
5	Grade 5
6	Grade 6
7	Grade 7
8	Grade 8
A1HS	High School Algebra I
GHS	High School Geometry
A2HS	High School Algebra II – Mathematics III
4HSTP	High School Trigonometry/Pre-calculus – Mathematics IV
ASHS	Applied Statistics
TMS	Transition Mathematics for Seniors
AMM	Advanced Mathematical Modeling
C	Calculus
FAM	Financial Algebra/Mathematics
QR	Quantitative Reasoning
PS	Statistics
IMA	Introduction to Mathematical Applications

*Note: High School Mathematics I and High School Mathematics II standards can be found in Appendix A, which includes a chart that shows the Algebra I and Geometry standard alignment for those courses.

MATHEMATICS

The Standards begin with eight Mathematical Habits of Mind (MHM). The West Virginia College- and Career-Readiness Standards for Mathematics define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. What does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are accessible using mathematical tasks of sufficient richness.

Mathematics: Mathematical Habits of Mind (MHM)

The Mathematical Habits of Mind (MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize - to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents; and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and - if there is a flaw in an argument - explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols

they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms interact when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the MHM to the Standards for Mathematical Content
The MHM describe ways in which developing students of mathematics increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the MHM to mathematical content in mathematics instruction.

Mathematics – Kindergarten

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in kindergarten will focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. The skill progressions begin in kindergarten as foundational understanding of numeracy. The following chart represents the mathematical understandings that will be developed in kindergarten:

Counting and Cardinality	**Operations and Algebraic Thinking**
• Count objects to tell how many there are by ones and by tens. • Write numbers from 0 to 20. • Compare two groups of objects to tell which group, if either, has more; compare two written numbers to tell which is greater.	• Understand addition as putting together and adding to. • Understand subtraction as taking apart and taking from. • Fluently (efficiently, flexibly, and accurately) add and subtract very small numbers (e.g., 3 + 1). • Recognize Patterns.
Number and Operations in Base Ten	**Measurement and Data**
• Act out addition and subtraction word problems and draw diagrams to represent them. • Add with a sum of 10 or less; subtract from a number 10 or less; and solve addition and subtraction word problems. • Group objects by tens and ones (e.g., 1 group of 10 and 3 ones makes 13).	• Describe and compare objects as longer, shorter, larger, smaller, etc. • Classify objects and count the number of objects in each category. • Identify coins and count pennies.
Geometry	
• Name shapes correctly regardless of orientation or size (e.g., a square oriented as a "diamond" is still a square).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Counting and Cardinality	
Know number names and the count sequence.	Standards 1-3
Count to tell the number of objects.	Standards 4-5
Compare numbers.	Standards 6-7
Operations and Algebraic Thinking	
Understand addition as putting together and	Standards 8-12

adding to and understand subtraction as taking apart and taking from.	
Recognize Patterns.	Standard 13
Number and Operations in Base Ten	
Work with numbers 11-19 to gain foundations for place value.	Standard 14
Measurement and Data	
Describe and compare measurable attributes.	Standards 15-16
Classify objects and count the number of objects in each category.	Standard 17
Work with money.	Standards 18-19
Geometry	
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	Standards 20-22
Analyze, compare, create, and compose shapes.	Standards 23-25

Counting and Cardinality

Cluster	**Know number names and the count sequence.**
M.K.1	Count to 100 by ones and by tens.
M.K.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
M.K.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Cluster	**Count to tell the number of objects.**
M.K.4	Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted and the number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.
M.K.5	Count to answer questions (e.g., "How many?") about as many as 20 things arranged in a line, a rectangular array, a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Cluster	**Compare numbers.**
M.K.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g., by using matching and counting strategies).
M.K.7	Compare and order two numbers between 0-20 presented as written numerals.

Operations and Algebraic Thinking

Cluster	Understand addition as putting together and adding to and understand subtraction as taking apart and taking from.
M.K.8	Represent addition and subtraction with strategies using objects, fingers, mental images, drawings, sounds (e.g., claps), and acting out situations, verbal explanations, expressions, and equations.
M.K.9	Solve addition and subtraction word problems and add and subtract within 10 by using objects or drawings to represent the problem.
M.K.10	Decompose numbers less than or equal to 10 into pairs in more than one way by using objects or drawings and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
M.K.11	For any number from 1 to 9, find the number that makes 10 when added to the given number by using objects or drawings, and record the answer with a drawing or equation.
M.K.12	Fluently (efficiently, flexibly, and accurately) add and subtract within 5 using various strategies.

Cluster	Recognize patterns.
M.K.13	Recognize and create recognizable patterns using colors, shapes, sizes, and sounds with support and guidance.

Number and Operations in Base Ten

Cluster	Work with numbers 11-19 to gain foundations for place value.
M.K.14	Compose and decompose numbers from 11 to 19 into ten ones and larger two-digit numbers by using objects or drawings and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones (one ten) and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data

Cluster	Describe and compare measurable attributes.
M.K.15	Describe measurable attributes of objects, such as length or weight and describe several measurable attributes of a single object.
M.K.16	Directly compare two objects with a measurable attribute in common, to see which object has “more of” or “less of” the attribute and describe the difference.

Cluster	Classify objects and count the number of objects in each category.
M.K.17	Classify objects into given categories, count the numbers of objects in each category, and sort the categories by count. Category counts should be limited to less than or equal to 10. (e.g., Identify coins and sort them into groups of 5s or 10s.)

Cluster	Work with money.
M.K.18	Identify coins: penny, nickel, dime, quarter.
M.K.19	Count pennies to 20.

Geometry

Cluster	**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).**
M.K.20	Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind and next to.
M.K.21	Correctly name shapes regardless of their orientations or overall size.
M.K.22	Using real-life objects, identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Cluster	**Analyze, compare, create, and compose shapes.**
M.K.23	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/ “corners”), and other attributes (e.g., having sides of equal length). Instructional Note: Student focus should include real-world shapes.
M.K.24	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
M.K.25	Compose simple shapes to form larger shapes (e.g., “Can these two triangles, with full sides touching, join to make a rectangle?”).

Mathematics – Grade 1

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in the first grade will focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as repeating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from kindergarten, the following chart represents the mathematical understandings that will be developed in first grade:

Operations and Algebraic Thinking	**Number and Operations in Base Ten**
• Solve addition and subtraction word problems in situations of adding to, taking from, putting together, taking apart, and comparing (e.g., a taking from situation would be: "Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?"). • Add fluently (efficiently, flexibly, and accurately) with a sum of 10 or less, and accurately subtract from a number 10 or less (e.g., 2 + 5, 7 – 5). • Understanding the relationship between addition and subtraction.	• Understand what the digits mean in two-digit numbers (place value). • Compare and order numbers based on place value. • Use understanding of place value and properties of operations to add and subtract (e.g., 38 + 5, 29 + 20, 64 + 27, 80 – 50). • Identify the value of coins and use dimes and pennies to model numbers to 100. • Skip count by ones, twos, fives, and tens.
Measurement and Data	**Geometry**
• Measure lengths of objects by using a shorter object as a unit of length. • Tell and write time.	• Make composite shapes by joining shapes together and dividing circles and rectangles into halves or fourths. • Create recognizable patterns following a given rule.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Operations and Algebraic Thinking	
Represent and solve problems involving addition and subtraction.	Standards 1-2
Understand and apply properties of operations and the relationship between addition and subtraction.	Standards 3-4

Add and subtract within 20.	Standards 5-6
Work with addition and subtraction equations.	Standards 7-8
Number and Operations in Base Ten	
Extend the counting sequence.	Standard 9
Understand place value.	Standards 10-11
Use place value understanding and properties of operations to add and subtract.	Standards 12-14
Measurement and Data	
Measure lengths indirectly and by iterating length units.	Standards 15-16
Work with time and money.	Standards 17-18
Represent and interpret data.	Standard 19
Geometry	
Reason with shapes and their attributes.	Standards 20-23

Operations and Algebraic Thinking

Cluster	**Represent and solve problems involving addition and subtraction.**
M.1.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).
M.1.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).

Cluster	**Understand and apply properties of operations and the relationship between addition and subtraction.**
M.1.3	Apply properties of operations as strategies to add and subtract (e.g., If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known: Commutative Property of Addition. To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12: Associative Property of Addition). Instructional Note: Students need not use formal terms for these properties.
M.1.4	Understand subtraction as an unknown-addend problem (e.g., subtract 10 – 8 by finding the number that makes 10 when added to 8).

Cluster	**Add and subtract within 20.**
M.1.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2, by counting backwards 3 to subtract 3).
M.1.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10 and use strategies such as · counting on; · making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); · decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); · using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8= 4); and · creating equivalent but easier or known sums (e.g., adding 6 +7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Cluster	Work with addition and subtraction equations.
M.1.7	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false (e.g., Which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2). Recognize the difference between an expression (3 + 5) and an equation (3 + 5=8).
M.1.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers (e.g., Determine the unknown number that makes the equation true in each of the equations. 8 + ? = 11, 5 = ? – 3, 6 + 6 = ?).

Number and Operations in Base Ten

Cluster	Extend the counting sequence.
M.1.9	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. Skip count to 120 by 2's. Skip count to 120 by 5's and 10's.

Cluster	Understand place value.
M.1.10	Understand the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a "ten." (e.g., A group of ten pennies is equivalent to a dime.) b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight or nine tens (and 0 ones).
M.1.11	Compare and order two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Cluster	Use place value understanding and properties of operations to add and subtract.
M.1.12	Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones, and sometimes it is necessary to compose a ten.
M.1.13	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count and explain the reasoning used.
M.1.14	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences) using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used.

Measurement and Data

Cluster	Measure lengths indirectly and by iterating length units.
M.1.15	Order three objects by length and compare the lengths of two objects indirectly by using

	a third object.
M.1.16	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Instructional Note: Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Cluster	**Work with time and money.**
M.1.17	Tell and write time in hours and half-hours using analog and digital clocks.
M.1.18	Identify the value of coins and use dimes and pennies to model the relationship between money and place value (e.g., exchange 10 pennies for 1 dime or exchange 10 dimes for 1 dollar).

Cluster	**Represent and interpret data.**
M.1.19	Organize, represent, interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category and how many more or less are in one category than in another.

Geometry

Cluster	**Reason with shapes and their attributes.**
M.1.20	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, and/or overall size); build and draw shapes to possess defining attributes.
M.1.21	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and compose new shapes from the composite shape. Instructional Note: Students do not need to learn formal names such as, “right rectangular prism.”
M.1.22	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths and quarters and use the phrases half of, fourth of and quarter of. Describe the whole as two of, or four of the shares and understand for these examples that decomposing into more equal shares creates smaller shares.
M.1.23	Create a recognizable pattern following a given rule, using colors, shapes, sizes, and sounds.

Mathematics – Grade 2

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in the second grade will focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from first grade, the following chart represents the mathematical understandings that will be developed in second grade:

Operations and Algebraic Thinking	Number and Operations in Base Ten
• Solve challenging addition and subtraction word problems with one or two steps (e.g., a "one-step" problem would be: "Lucy has 23 fewer apples than Julie. Julie has 47 apples. How many apples does Lucy have?"). • Fluently add with a sum of 20 or less (e.g., 11 + 8); fluently subtract from a number 20 or less (e.g., 16 – 9); and know all sums of one-digit numbers from memory by the end of the year. • Work with equal groups of objects to gain foundations for multiplication. • Analyze number patterns.	• Understand what the digits mean in three-digit numbers (place value). • Use an understanding of place value to add and subtract three-digit numbers (e.g., 811 – 367); add and subtract two-digit numbers fluently (e.g., 77 – 28).
Measurement and Data	**Geometry**
• Solve addition and subtraction word problems involving length (e.g., "The pen is 2 cm longer than the pencil. If the pencil is 7 cm long, how long is the pen?"). • Tell time. • Count money.	• Build, draw, and analyze 2-D and 3-D shapes to develop foundations for area, volume, and geometry in later grades. • Divide shapes into equal shares to build the foundations for fractions in later grades.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Operations and Algebraic Thinking	
Represent and solve problems involving addition and subtraction.	Standard 1
Solve problems involving addition and subtraction and identify and explain patterns in arithmetic.	Standards 2-3
Work with equal groups of objects to gain	Standards 4-5

foundations for multiplication.	
Number and Operations in Base Ten	
Understand place value.	Standards 6-9
Use place value understanding and properties of operations to add and subtract.	Standards 10-14
Measurement and Data	
Measure and estimate lengths in standard units.	Standards 15-18
Relate addition and subtraction to length.	Standards 19-20
Work with time and money.	Standards 21-22
Represent and interpret data.	Standards 23-24
Geometry	
Reason with shapes and their attributes.	Standards 25-27

Operations and Algebraic Thinking

Cluster	**Represent and solve problems involving addition and subtraction.**
M.2.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).

Cluster	**Solve problems involving addition and subtraction and identify and explain patterns in arithmetic.**
M.2.2	Fluently (efficiently, flexibly, and accurately) add and subtract within 20 using mental strategies and by end of Grade 2, know from memory all sums of two one-digit numbers by using various strategies.
M.2.3	Analyze a number pattern to determine the rule: add 2, add 3, add 5, or add 10.

Cluster	**Work with equal groups of objects to gain foundations for multiplication.**
M.2.4	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
M.2.5	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns to describe the arrays; write expressions to describe the array and equations to express the total as a sum of equal addends.

Number and Operations in Base Ten

Cluster	**Understand place value.**
M.2.6	Understand that the three digits of a three-digit number represent amounts of hundreds, tens and ones (e.g., 706 equals 7 hundreds, 0 tens and 6 ones). Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens – called a "hundred." b. Numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight or nine hundreds, and 0 tens and 0 ones.
M.2.7	Count within 1000 and skip-count by 5s, 10s and 100s.
M.2.8	Read and write numbers to 1000 using base-ten numerals, number names and expanded

	form.
M.2.9	Compare two three-digit numbers based on meanings of the hundreds, tens and ones digits, using >, = and < symbols to record the results of comparisons. Order numbers based on place value.

Cluster	**Use place value understanding and properties of operations to add and subtract.**
M.2.10	Fluently add and subtract within 100 using strategies based on place value, properties of operations and/or the relationship between addition and subtraction.
M.2.11	Add up to four two-digit numbers using strategies based on place value and properties of operations.
M.2.12	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones and sometimes it is necessary to compose or decompose tens or hundreds.
M.2.13	Mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number 100-900.
M.2.14	Explain why addition and subtraction strategies work, using place value and the properties of operations. Instructional Note: Explanations may be supported by drawing or objects.

Measurement and Data

Cluster	**Measure and estimate lengths in standard units.**
M.2.15	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
M.2.16	Measure the length of an object twice, using length units of different lengths for the two measurements, describe how the two measurements relate to the size of the unit chosen.
M.2.17	Estimate lengths using units of inches, feet, centimeters, and meters.
M.2.18	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Cluster	**Relate addition and subtraction to length.**
M.2.19	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units (e.g., by using drawings, such as drawings of rulers), and equations with a symbol for the unknown number to represent the problem.
M.2.20	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2... and represent whole-number sums and differences within 100 on a number line diagram.

Cluster	**Work with time and money.**
M.2.21	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
M.2.22	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately (e.g., If you have 2 dimes and 3 pennies, how many cents do

	you have?).

Cluster	Represent and interpret data.
M.2.23	Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
M.2.24	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Geometry

Cluster	Reason with shapes and their attributes.
M.2.25	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces (sizes are compared directly or visually, not compared by measuring). Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
M.2.26	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
M.2.27	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Mathematics – Grade 3

West Virginia teachers who provide mathematics instruction must integrate-content standards with the MHM. Students in the third grade will focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from second grade, the following chart represents the mathematical understandings that will be developed in third grade:

Operations and Algebraic Thinking	**Number and Operations in Base Ten**
• Understand how to multiply and divide numbers up to 10 x 10 fluently (efficiently, flexibly, and accurately). • Solve word problems using addition, subtraction, multiplication, and division. • Begin to multiply numbers with more than one digit (e.g., multiplying 9 × 80).	• Understand place value and properties of operations to perform multi-digit arithmetic, such as 10 x 2, 50 x 3, and 40 x 7. • Compare and order numbers based on place value. • Identify an arithmetic pattern.
Number and Operations- Fractions	**Measurement and Data**
• Understand fractions and relate them to the familiar system of whole numbers (e.g., recognizing that 3/1 and 3 are the same number).	• Measure and estimate weights and liquid volumes and solve word problems involving these quantities. • Tell time and write time to the nearest minute. • Recognize area as a quality of two-dimensional regions. • Understand that rectangular arrays can be broken into identical rows or into identical columns. By breaking rectangles into rectangular arrays of squares, students connect area to multiplication and explain how multiplication is used to determine the area of a rectangle.
Geometry	
• Reason about shapes (e.g., all squares are rectangles but not all rectangles are squares). • Find areas of shapes, and relate area to multiplication (e.g., why is the number of square feet for a 9-foot by 7-foot room given by the product 9 × 7?). • Understand the connection between equal parts of a shape being a unit of the whole.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Operations and Algebraic Thinking	
Represent and solve problems involving multiplication and division.	Standards 1-4
Understand properties of multiplication and the	Standards 5-6

relationship between multiplication and division.	
Multiply and divide within 100.	Standard 7
Solve problems involving the four operations and identify and explain patterns in arithmetic.	Standards 8-9
Number and Operations in Base Ten	
Use place value and properties of operations to perform multi-digit arithmetic.	Standards 10-14
Number and Operations – Fractions	
Develop an understanding of fractions as numbers.	Standards 15-17
Measurement and Data	
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	Standards 18-19
Represent and interpret data.	Standards 20-21
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	Standards 22-24
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	Standard 25
Geometry	
Reason with shapes and their attributes.	Standards 26-27

Operations and Algebraic Thinking

Cluster	**Represent and solve problems involving multiplication and division.**
M.3.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each (e.g., describe context in which a total number of objects can be expressed as 5 × 7).
M.3.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each (e.g., describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8).
M.3.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
M.3.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers (e.g., determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 =?).

Cluster	**Understand properties of multiplication and the relationship between multiplication and division.**
M.3.5	Apply properties of operations as strategies to multiply and divide (e.g., if 6 × 4 = 24 is known, then 4 × 6 = 24 is also known: Commutative Property of Multiplication; 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30:

	Associative Property of Multiplication; knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56: Distributive Property). Instructional Note: Students should focus on conceptual understanding not use formal terms for these properties.
M.3.6	Understand division as an unknown-factor problem (e.g., find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8).

Cluster	**Multiply and divide within 100.**
M.3.7	Fluently (efficiently, flexibly, and accurately) multiply and divide within 100 using strategies such as the relationship between multiplication and division and the properties of operations. By the end of Grade 3, know the multiplication table (facts) within 100 (0s-10s) efficiently.

Cluster	**Solve problems involving the four operations and identify and explain patterns in arithmetic.**
M.3.8	Solve two-step word problems using the four operations, represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. Instructional Note: This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
M.3.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain those using properties of operations (e.g., observe that 4 times a number is always even and explain why 4 times a number can be decomposed into two equal addends).

Number and Operations in Base Ten

Cluster	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**
M.3.10	Read and write numbers to 10,000 using standard form, word form, and expanded form.
M.3.11	Compare two four-digit numbers based on meanings of the thousands, hundreds, tens, and ones digits using >, = and < symbols to record the results of the comparisons. Order numbers based on place value.
M.3.12	Use place value understanding to round whole numbers to the nearest 10 or 100.
M.3.13	Fluently (efficiently, flexibly, and accurately) add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
M.3.14	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations- Fractions

Cluster	**Develop understanding of fractions as numbers.**
M.3.15	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size

	1/b. Instructional Note: Fractions in this standard are limited to denominators of 2, 3, 4, 6, and 8.
M.3.16	Understand a fraction as a number on the number line and represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line (e.g., given that b parts is 4 parts, then 1/b represents ¼; students partition the number line into fourths and locate 1/4 on the number line). b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line (e.g., given that a/b represents 3/4 or 6/4, students partition the number line into fourths and represent these fractions accurately on the same number line; students extend the number line to include the number of wholes required for the given fractions). Instructional Note: Fractions in this standard are limited to denominators of 2, 3, 4, 6, and 8.
M.3.17	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line. b. Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model). c. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers (e.g., express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram). d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, = or < and justify the conclusions (e.g., by using a visual fraction model). Instructional Note: Fractions in this standard are limited to denominators of 2, 3, 4, 6, and 8.

Measurement and Data

Cluster	**Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**
M.3.18	Tell and write time to the nearest minute, measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., by representing the problem on a number line diagram).
M.3.19	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg) and liters (l). Add, subtract, multiply or divide to solve one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale) to represent the problem. Instructional Note: Exclude compound units such as cm^3 and finding the geometric volume of a container.

Cluster	Represent and interpret data.
M.3.20	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs (e.g., draw a bar graph in which each square might represent 5 pets).
M.3.21	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves or quarters.

Cluster	Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
M.3.22	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by b unit squares is said to have an area of b square units.
M.3.23	Measure areas by counting unit squares (square cm, square m, square in, square ft. and improvised units).
M.3.24	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive and find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Cluster	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
M.3.25	Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry

Cluster	Reason with shapes and their attributes.
M.3.26	Understand that shapes in distinct categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides) and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

M.3.27	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as ¼ or the area of the shape.

Mathematics – Grade 4

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in the fourth grade will focus on three critical areas: (1) developing understanding with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from third grade, the following chart represents the mathematical understandings that will be developed in fourth grade:

<table>
<tr><th>Operations and Algebraic Thinking</th><th>Number and Operations in Base Ten</th></tr>
<tr><td>• Use whole-number arithmetic to solve word problems, including problems with remainders and problems with measurements.
• Add and subtract whole numbers efficiently and accurately using the standard algorithm.
• Multiply and divide multi-digit numbers by a one-digit number (e.g., multiplying 1,638 x 7 and dividing 6,966 by 6).
• Multiply two 2-digit numbers.
• Gain familiarity with factors and multiples.
• Generate and analyze patterns.</td><td>• Generalize place value understanding for multi-digit whole numbers.
• Compare and order numbers based on place value.
• Use place value understanding and properties of operations to perform multi-digit arithmetic.</td></tr>
<tr><th>Number and Operations- Fractions</th><th>Measurement and Data</th></tr>
<tr><td>• Use equivalent fractions to understand and order fractions (e.g., recognize that 1⁄4 is less than 3⁄8 because 2⁄8 is less than 3⁄8).
• Add and subtract fractions and mixed numbers with like denominators (such as 2 3⁄4 – 1 1⁄4) and solve related word problems.
• Multiply fractions by whole numbers.
• Understand and compare simple decimals in terms of fractions (e.g., rewriting 0.62 as 62⁄100).</td><td rowspan="2">• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
• Represent and interpret data.
• Measure angles and understand concepts of angles.</td></tr>
<tr><th>Geometry</th></tr>
<tr><td>• Draw and identify lines and angles and classify shapes by properties of their lines and angles.
• Measure angles and find unknown angles in a diagram.</td></tr>
</table>

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Operations and Algebraic Thinking	
Use the four operations with whole numbers to solve problems.	Standards 1-3
Gain familiarity with factors and multiples.	Standard 4
Generate and analyze patterns.	Standard 5
Number and Operations in Base Ten	
Generalize place value understanding for multi-digit whole numbers.	Standards 6-8
Use place value understanding and properties of operations to perform multi-digit arithmetic.	Standards 9-11
Number and Operations- Fractions	
Extend understanding of fraction equivalence and ordering.	Standards 12-13
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	Standards 14-15
Understand decimal notation for fractions and compare decimal fractions.	Standards 16-18
Measurement and Data	
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	Standards 19-21
Represent and interpret data.	Standard 22
Geometric measurement: understand concepts of angle and measure angles.	Standards 23-25
Geometry	
Draw and identify lines angles and classify shapes by properties of their lines and angles.	Standards 26-28

Operations and Algebraic Thinking

Cluster	**Use the four operations with whole numbers to solve problems.**
M.4.1	Interpret a multiplication equation as a comparison of two expressions (e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication expressions and equations.
M.4.2	Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem) and distinguish multiplicative comparison from additive comparison.
M.4.3	Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the

	unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Cluster	Gain familiarity with factors and multiples.
M.4.4	Find all factor pairs for a whole number in the range 1–100, recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Cluster	Generate and analyze patterns.
M.4.5	Generate a number pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself (e.g., given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers; explain informally why the numbers will continue to alternate in this way).

Number and Operations in Base Ten

Cluster	Generalize place value understanding for multi-digit whole numbers.
M.4.6	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right (e.g., recognize that 700 ÷ 70 = 10 by applying concepts of place value and division).
M.4.7	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, = and < symbols to record the results of comparisons. Order numbers based on place value.
M.4.8	Use place value understanding to round multi-digit whole numbers to any place.

Cluster	Use place value understanding and properties of operations to perform multi-digit arithmetic.
M.4.9	Fluently (efficiently and accurately) add and subtract multi-digit whole numbers using the standard algorithm.
M.4.10	Multiply a whole number of up to four digits by a one-digit whole number, multiply two two-digit numbers, using strategies based on place value and the properties of operations and illustrate and explain the calculation by using equations, rectangular arrays, area models, and/or partial products.
M.4.11	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, area models, and/or partial quotients.

Number and Operations- Fractions

Cluster	Extend understanding of fraction equivalence and ordering.
M.4.12	Explain why a fraction a/b is equivalent to another fraction by using visual fraction models, with attention to how the number and size of the parts differ even though the

	two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
M.4.13	Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or common numerators, or by comparing to a benchmark fraction such as ½). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, = or <, and justify the conclusions by using a visual fraction model.

Cluster	**Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**
M.4.14	Understand the fraction a/b, with a > 1, as the sum of a of the fractions 1/b. a. Add and subtract fractions with like denominators. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions by using a visual fraction model (e.g., 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8). c. Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction greater than one and/or by using properties of operations and the relationship between addition and subtraction. Identify the two whole numbers a mixed number is between. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators by using visual fraction models and equations to represent the problem.
M.4.15	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of 1/b, (e.g., use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4)). b. Understand a multiple of a/b as a multiple of 1/b and use this understanding to multiply a fraction by a whole number (e.g., use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. In general, n × (a/b) = (n × a)/b). c. Solve word problems involving multiplication of a fraction by a whole number by using visual fraction models and equations to represent the problem (e.g., If each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?).

Cluster	**Understand decimal notation for fractions and compare decimal fractions.**
M.4.16	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100 (e.g., express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100). Instructional Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

M.4.17	Use decimal notation for fractions with denominators 10 or 100 (e.g., rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram).
M.4.18	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, = or <, and justify the conclusions by using a visual model.

Measurement and Data

Cluster	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**
M.4.19	Know relative sizes of measurement units within a system of units, including the metric system (km, m, cm; kg, g; l, ml), the customary system (lb., oz.), and time (hr., min., sec.). Within one system of measurement, express measurements in a larger unit in terms of a smaller unit. Express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table (e.g., know that 1 ft is 12 times as long as 1 in; express the length of a 4 ft snake as 48 in; generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...).
M.4.20	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
M.4.21	Apply the area and perimeter formulas for rectangles in real-world and mathematical problems by viewing the area formula as a multiplication equation with an unknown factor (e.g., find the width of a rectangular room given the area of the flooring and the length).

Cluster	**Represent and interpret data.**
M.4.22	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions with like denominators by using information presented in line plots (e.g., from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection).

Cluster	**Geometric measurement: understand concepts of angles and measure angles.**
M.4.23	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used for measuring angles. b. An angle that turns through b one-degree angles is said to have an angle measure of b degrees.
M.4.24	Measure angles in whole-number degrees using a protractor and sketch angles of specified measure.
M.4.25	Recognize angle measure as additive. When an angle is decomposed into non-

	overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure).

Geometry

Cluster	**Draw and identify lines and angles and classify shapes by properties of their lines and angles.**
M.4.26	Draw points, lines, line segments, rays, angles (right, acute, obtuse) and perpendicular and parallel lines. Identify these in two-dimensional figures.
M.4.27	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles.
M.4.28	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Mathematics – Grade 5

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in the fifth grade will focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; (3) developing an understanding of volume. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in fifth grade will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from fourth grade, the following chart represents the mathematical understandings that will be developed in fifth grade:

<table>
<tr><th>Operations and Algebraic Thinking</th><th>Number and Operations in Base Ten</th></tr>
<tr><td>• Write and interpret numerical expressions.
• Analyze mathematical patterns and relationships.</td><td>• Understand the place value system.
• Generalize the place-value system to include decimals and calculate with decimals to the hundredths place (two places after the decimal).
• Multiply whole numbers fluently (efficiently and accurately) by using the standard algorithm, for example 1,638 × 753, and divide whole numbers by using a strategy based on place value and properties, for example, dividing 6,971 by 63.</td></tr>
<tr><th>Number and Operations- Fractions</th><th>Measurement and Data</th></tr>
<tr><td>• Add and subtract fractions and mixed numbers with like and unlike denominators (e.g., 2 1⁄4 – 1 1⁄3) and solve word problems of this kind.
• Multiply fractions; divide fractions in simple cases; and solve related word problems (e.g., find the area of a rectangle with fractional side lengths; determine how many 1⁄3-cup servings are in 2 cups of raisins; or if 3 people share 1⁄2 pound of chocolate equally).</td><td rowspan="3">• Convert like measurement units within a given measurement system.
• Make a line plot to display a data set with fractional units of measure and interpret the data to solve problems.
• Geometric measurement: Understand the concept of volume and solve word problems that involve volume.</td></tr>
<tr><th>Geometry</th></tr>
<tr><td>• Graph points in quadrant 1 of the coordinate plane to solve real-world and mathematical problems.
• Classify two-dimensional figures into categories based on their properties.</td></tr>
</table>

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Operations and Algebraic Thinking	
Write and interpret numerical expressions.	Standards 1-2
Analyze patterns and relationships.	Standard 3
Number and Operations in Base Ten	
Understand the place value system.	Standards 4-7
Perform operations with multi-digit whole numbers and with decimals to hundredths.	Standards 8-10
Number and Operations- Fractions	
Use equivalent fractions as a strategy to add and subtract fractions.	Standards 11-12
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	Standards 13-17
Measurement and Data	
Convert like measurement units within a given measurement system.	Standard 18
Represent and interpret data.	Standard 19
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	Standards 20-22
Geometry	
Graph points on the coordinate plane to solve real-world and mathematical problems.	Standards 23-24
Classify two-dimensional figures into categories based on their properties.	Standards 25-26

Operations and Algebraic Thinking

Cluster	**Write and interpret numerical expressions.**
M.5.1	Use parentheses or brackets in numerical expressions and evaluate expressions with these symbols.
M.5.2	Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them (e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7); recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product).

Cluster	**Analyze patterns and relationships**.
M.5.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane (e.g., given the rule "Add 3" and the starting number 0 and given the rule "Add 6" and the starting

	number 0, generate terms in the resulting sequences and observe that the terms in one sequence are twice the corresponding terms in the other sequence; explain informally why this is so).

Number and Operations in Base Ten

Cluster	**Understand the place value system.**
M.5.4	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
M.5.5	Explain how the value of a multi-digit number, including decimals, is changed when the number is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
M.5.6	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names and expanded form (e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000)). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, = and < symbols to record the results of comparisons.
M.5.7	Use place value understanding to round multi-digit numbers, including decimals, to any place.

Cluster	**Perform operations with multi-digit whole numbers and with decimals to hundredths.**
M.5.8	Fluently (efficiently and accurately) multiply multi-digit whole numbers using the standard algorithm.
M.5.9	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, area models, and/or partial quotients.
M.5.10	Add, subtract, multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between related operations, relate the strategy to a written method and explain the reasoning used.

Number and Operations - Fractions

Cluster	**Use equivalent fractions as a strategy to add and subtract fractions.**
M.5.11	Add and subtract fractions with unlike denominators by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (e.g., 2/3 + 5/4 = 8/12 + 15/12 = 23/12). Instructional Note: In general, a/b + c/d = (ad + bc)/bd.
M.5.12	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (e.g., recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2).

Cluster	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**
M.5.13	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models or equations to represent the problem (e.g., interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3 and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4).
M.5.14	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) × q as a number of parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b (e.g., use a visual fraction model to show (2/3) × 4 = 8/3 and create a story context for this equation; do the same with (2/3) × (4/5) = 8/15). Instructional Note: In general, (a/b) × (c/d) = ac/bd. b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.
M.5.15	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1.
M.5.16	Solve real-world problems involving multiplication of fractions and mixed numbers by using visual fraction models or equations to represent the problem.
M.5.17	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Instructional Note: Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade. a. Interpret division of a unit fraction by a non-zero whole number and compute such quotients (e.g., create a story context for (1/3) ÷ 4 and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3). b. Interpret division of a whole number by a unit fraction and compute such quotients (e.g., create a story context for 4 ÷ (1/5) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4). c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions by using visual fraction models and equations to represent the problem (e.g., How much chocolate will

	each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?).

Measurement and Data

Cluster	**Convert like measurement units within a given measurement system.**
M.5.18	Convert among different-sized standard measurement units within a given measurement system, both customary and metric, (e.g., convert 5 cm to 0.05 m) and use these conversions in solving multi-step, real-world problems.

Cluster	**Represent and interpret data.**
M.5.19	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots (e.g., given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally).

Cluster	**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**
M.5.20	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using b unit cubes is said to have a volume of b cubic units.
M.5.21	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
M.5.22	Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). b. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive and find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Geometry

Cluster	**Graph points on the coordinate plane to solve real-world and mathematical problems.**
M.5.23	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines, the origin, arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its

	coordinates. Understand that the first number indicates how far to travel from the origin in the direction of the horizontal axis (x-axis) and the second number indicates how far to travel in the direction of the vertical axis (y-axis), with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
M.5.24	Represent real-world mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

Cluster	**Classify two-dimensional figures into categories based on their properties.**
M.5.25	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category (e.g., all rectangles have four right angles and squares are rectangles, so all squares have four right angles).
M.5.26	Classify two-dimensional figures in a hierarchy based on properties.

Mathematics – Grade 6

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in the sixth grade will focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting and using expressions and equations; and (4) developing understanding of statistical thinking. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in sixth grade will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from fifth grade, the following chart represents the mathematical understandings that will be developed in sixth grade:

<table>
<tr><th>Ratios and Proportional Reasoning</th><th>The Number System</th></tr>
<tr><td>• Understand ratios and rates, and solve problems involving proportional relationships (e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours?).</td><td>• Divide fractions and solve related word problems (e.g., How wide is a rectangular strip of land with length 3/4 mile and area 1/2 square mile?).
• Perform operations with multi-digit decimals and solve related word problems.
• Use positive and negative numbers together to describe quantities; understand the ordering and absolute values of positive and negative numbers.</td></tr>
<tr><th>Expressions and Equations</th><th>Geometry</th></tr>
<tr><td>• Work with variables and expressions by generalizing the way numbers work (e.g., when adding numbers, the order doesn't matter, so x + y = y + x; likewise, properties of addition and multiplication can be used to rewrite 24x + 18y as 6(4x + 3y), or y + y + y as 3y).
• Write equations to solve word problems and describe relationships between quantities (e.g., the distance D traveled by a train in time T might be expressed by an equation D = 85T, where D is in miles and T is in hours).</td><td rowspan="3">• Reason about relationships between shapes to determine area, surface area, and volume.</td></tr>
<tr><th>Statistics and Probability</th></tr>
<tr><td>• Create graphical representations of data and reason about statistical distributions.</td></tr>
</table>

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Ratios and Proportional Relationships	
Understand ratio concepts and use ratio reasoning to solve problems.	Standards 1-3
The Number System	
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	Standard 4
Compute fluently with multi-digit numbers and find common factors and multiples.	Standards 5-7
Apply and extend previous understandings of numbers to the system of rational numbers.	Standards 8-11
Expressions and Equations	
Apply and extend previous understandings of arithmetic to algebraic expressions.	Standards 12-15
Reason about and solve one-variable equations and inequalities.	Standards 16-19
Represent and analyze quantitative relationships between dependent and independent variables.	Standard 20
Geometry	
Solve real-world and mathematical problems involving area, surface area, and volume.	Standards 21-24
Statistics and Probability	
Develop understanding of statistical variability.	Standards 25-27
Summarize and describe distributions.	Standards 28-29

Ratios and Proportional Relationships

Cluster	Understand ratio concepts and use ratio reasoning to solve problems.
M.6.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities (e.g., "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes.").
M.6.2	Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$ and use rate language in the context of a ratio relationship (e.g., "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."). Instructional Note: Expectations for unit rates in this grade are limited to non-complex fractions.
M.6.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number

	measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed (e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?). c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System

Cluster	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**
M.6.4	Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions by using visual fraction models and equations to represent the problem (e.g., create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. [In general, (a/b) ÷ (c/d) = ad/bc.] How much chocolate will each person get if 3 people share 1/2 lb or 1 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup or 5/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area ½ square mi?).

Cluster	**Compute fluently with multi-digit numbers and find common factors and multiples.**
M.6.5	Fluently (efficiently and accurately) divide multi-digit numbers using the standard algorithm.
M.6.6	Fluently (efficiently and accurately) add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation.
M.6.7	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor (e.g., express 36 + 8 as 4 (9 + 2)).

Cluster	**Apply and extend previous understandings of numbers to the system of rational numbers.**
M.6.8	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
M.6.9	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the

	number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
M.6.10	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram (e.g., interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right). b. Write, interpret, and explain statements of order for rational numbers in real-world contexts (e.g., write –3° C > –7° C to express the fact that –3° C is warmer than –7° C). c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation (e.g., for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars). d. Distinguish comparisons of absolute value from statements about order (e.g., recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars).
M.6.11	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations

Cluster	**Apply and extend previous understandings of arithmetic to algebraic expressions.**
M.6.12	Write and evaluate numerical expressions involving whole-number exponents.
M.6.13	Write, read and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers (e.g., express the calculation, "Subtract y from 5" as 5 – y). b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity (e.g., describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms). c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order: Order of Operations (e.g., use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length s = 1/2).
M.6.14	Apply the properties of operations to generate equivalent expressions (e.g., apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y).

M.6.15	Identify when two expressions are equivalent; i.e., when the two expressions name the same number regardless of which value is substituted into them (e.g., the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for).

Cluster	**Reason about and solve one-variable equations and inequalities.**
M.6.16	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
M.6.17	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or depending on the purpose at hand, any number in a specified set.
M.6.18	Solve real-world and mathematical problems by writing and solving a. Equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. b. Inequalities of the form $x + p > q$, $x + p < q$, $px > q$, and $px < q$ for cases in which p, q, and x are all nonnegative rational numbers. Instructional Note: Symbols may include $<$, $>$, $\leq$, $\geq$, $\neq$, and $=$ and words may include, but are not limited to, more than, exactly, fewer, at least, at most, and exceeds.
M.6.19	Write and identify an inequality of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$, to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$, have infinitely many solutions; represent solutions of such inequalities on number line diagrams. Instructional Note: Symbols may include $<$, $>$, $\leq$, $\geq$, $\neq$, and $=$ and words may include, but are not limited to, more than, exactly, fewer, at least, at most, and exceeds.

Cluster	**Represent and analyze quantitative relationships between dependent and independent variables.**
M.6.20	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation (e.g., in a problem involving motion at constant speed, list and graph ordered pairs of distances and times; write the equation $d = 65t$ to represent the relationship between distance and time).

Geometry

Cluster	**Solve real-world and mathematical problems involving area, surface area, and volume.**
M.6.21	Find the area of right triangles, other triangles, special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
M.6.22	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional

	edge lengths in the context of solving real-world and mathematical problems.
M.6.23	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
M.6.24	Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability

Cluster	**Develop understanding of statistical variability.**
M.6.25	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers (e.g., "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages).
M.6.26	Through informal observation, understand that a set of data collected to answer a statistical question has a distribution which can be described by its center (mean/median), spread (range), and overall shape.
M.6.27	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number.

Cluster	**Summarize and describe distributions.**
M.6.28	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
M.6.29	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center to the shape of the data distribution and the context in which the data were gathered.

Mathematics – Grade 7

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in the seventh grade will focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions and working with two- and three-dimensional shapes to solve problems involving area, surface area and volume; and (4) drawing inferences about populations based on samples. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in seventh grade will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from sixth grade, the following chart represents the mathematical understandings that will be developed in seventh grade:

Ratios and Proportional Reasoning	**The Number System**
• Analyze proportional relationships (e.g., by graphing in the coordinate plane), and distinguish proportional relationships from other kinds of mathematical relationships (e.g., buying 10 times as many items will cost you 10 times as much, but taking 10 times as many aspirin will not lower your fever 10 times as much).	• Solve percent problems (e.g., tax, tips, and markups and markdowns). • Solve word problems that have a combination of whole numbers, fractions, and decimals (e.g., a woman making $25 per hour receives a 10% raise; she will make an additional 1⁄10 of his or her salary an hour, or $2.50, for a new salary of $27.50).
Expressions and Equations	**Geometry**
• Solve equations such as 1⁄2 (x – 3) = 3⁄4 quickly and accurately and write equations of this kind to solve word problems.	• Solve problems involving scale drawings, circle measurements, triangle attributes, and angle relationships.
Statistics and Probability	
• Use statistics and probability to draw inferences and make comparisons (e.g., deciding which candidate is likely to win an election based on a survey).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Ratios and Proportional Relationships	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	Standards 1-3
The Number System	
Apply and extend previous understandings of operations with fractions to add, subtract,	Standards 4-6

multiply, and divide rational numbers.	
Expressions and Equations	
Use properties of operations to generate equivalent expressions.	Standards 7-8
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	Standards 9-10
Geometry	
Draw, construct and describe geometrical figures and describe the relationships between them.	Standards 11-13
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	Standards 14-16
Statistics and Probability	
Use random sampling to draw inferences about a population.	Standards 17-18
Draw informal comparative inferences about two populations.	Standards 19-22
Investigate chance processes and develop, use, and evaluate probability models.	Standards 23-26

Ratios and Proportional Relationships

Cluster	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
M.7.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units (e.g., if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour).
M.7.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations (e.g., if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn). d. Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation. Focus special attention on the points (0,0) and (1,r) where r is the unit rate.
M.7.3	Use proportional relationships to solve multistep ratio and percent problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and/or percent error).

The Number System

Cluster	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
M.7.4	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0 (e.g., a hydrogen atom has 0 charge because its two constituents are oppositely charged). b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction, depending on whether q is positive or negative. (e.g., to add "p + q" on the number line, start at "0" and move to "p" then move |q| in the positive or negative direction depending on whether "q" is positive or negative). Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.
M.7.5	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
M.7.6	Solve real-world and mathematical problems involving the four operations with rational numbers. Instructional Note: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Expressions and Equations

Cluster	Use properties of operations to generate equivalent expressions.
M.7.7	Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients.
M.7.8	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related (e.g., a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05.").

Cluster	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
M.7.9	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies (e.g., if a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50; if you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation).
M.7.10	Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach (e.g., the perimeter of a rectangle is 54 cm; its length is 6 cm; what is its width?; an arithmetic solution similar to "54 – 6 – 6 divided by 2" may be compared with the reasoning involved in solving the equation $2w + 12 = 54$; an arithmetic solution similar to "54/2 – 6" may be compared with the reasoning involved in solving the equation $2(w + 6) = 54$). b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem (e.g., as a salesperson, you are paid $50 per week plus $3 per sale; this week you want your pay to be at least $100; write an inequality for the number of sales you need to make and describe the solutions).

Geometry

Cluster	Draw, construct and describe geometrical figures and describe the relationships between them.
M.7.11	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
M.7.12	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine the following: a. a unique triangle (e.g., three side measures satisfy the triangle inequality theorem), b. more than one triangle (e.g., given three angles whose sum is 180 degrees), or c. no triangle (e.g., angle sum is not 180 degrees or sum of the measures of two sides does not exceed the measure of the third side).
M.7.13	Describe the two-dimensional figures (face shapes) that result from slicing three-dimensional figures with cuts made parallel to, perpendicular to, or neither parallel nor

	perpendicular to the bases of right rectangular prisms and right rectangular pyramids.

Cluster	**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
M.7.14	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. Instructional Note: Include problems where students must determine area given circumference but not vice versa.
M.7.15	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
M.7.16	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability

Cluster	**Use random sampling to draw inferences about a population.**
M.7.17	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
M.7.18	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions (e.g., estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data; gauge how far off the estimate or prediction might be).

Cluster	**Draw informal comparative inferences about two populations.**
M.7.19	Given two data displays, distinguish measures of center and measures of variation.
M.7.20	Compare two numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and describing any overall pattern(s). d. Giving quantitative measures of variability (interquartile range (IQR), range, and/or mean absolute deviation (MAD)) and describing any striking deviations from the overall pattern(s). e. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
M.7.21	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability (e.g., the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable).

M.7.22	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations (e.g., decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book).

Cluster	**Investigate chance processes and develop, use, and evaluate probability models.**
M.7.23	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely and a probability near 1 indicates a likely event.
M.7.24	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency and predict the approximate relative frequency given the probability (e.g., when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times). Instructional Note: Provide adequate models for students to understand relative frequency is the experimental probability which when given enough trials will match the theoretical probability.
M.7.25	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events (e.g., if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected). b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process (e.g., find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down; do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?).
M.7.26	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events (e.g., use random digits as a simulation tool to approximate the answer to the question: if 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?).

Mathematics – Grade 8

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in the eighth grade will focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and inequalities; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity and congruence and understanding and applying the Pythagorean Theorem. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in eighth grade will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from seventh grade, the following chart represents the mathematical understandings that will be developed in eighth grade:

The Number System	**Expressions and Equations**
• Understand that every number has a decimal expansion and use these to compare the size of irrational numbers.	• Work with positive and negative exponents, square root and cube root symbols, and scientific notation (e.g., Evaluate $\sqrt{36} + 64$; estimate world population as 7×10^9). • Solve linear equations (e.g., $-x + 5(x + 1/3) = 2x - 8$); solve pairs of linear equations (e.g., $x + 6y = -1$ and $2x - 2y = 12$); and write equations to solve related word problems.
Functions	**Geometry**
• Understand slope and relating linear equations in two variables to lines in the coordinate plane. • Understand functions as rules that assign a unique output number to each input number; use linear functions to model relationships.	• Understand congruence and similarity using physical models, transparencies, or geometry software (e.g., given two congruent figures, show how to obtain one from the other by a sequence of rotations, translations, and/or reflections).
Statistics and Probability	
• Analyze statistical relationships by using a best-fit line (a straight line that models an association between two quantities).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

The Number System	
Know that there are numbers that are not rational and approximate them by rational numbers.	Standards 1-2

Expressions and Equations	
Work with radicals and integer exponents.	Standards 3-6
Understand the connections between proportional relationships, lines, and linear equations.	Standards 7-8
Analyze and solve linear equations, pairs of simultaneous linear equations, and linear inequalities in one variable.	Standards 9-13
Functions	
Define, evaluate, and compare functions.	Standards 14-16
Use functions to model relationships between quantities.	Standards 17-18
Geometry	
Understand congruence and similarity using physical models, transparencies, or geometry software.	Standards 19-23
Understand and apply the Pythagorean Theorem.	Standards 24-26
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	Standard 27
Statistics and Probability	
Investigate patterns of association in bivariate data.	Standards 28-31

The Number System

Cluster	**Know that there are numbers that are not rational and approximate them by rational numbers.**
M.8.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number. Instructional Note: A decimal expansion that repeats the digit 0 is often referred to as a "terminating decimal."
M.8.2	Apply approximations and properties of rational and irrational numbers to: a. Compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions such as π^2 (e.g., by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations). b. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Expressions and Equations

Cluster	**Work with radicals and integer exponents.**
M.8.3	Know and apply the properties of integer exponents to generate equivalent numerical expressions (e.g., $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$).

M.8.4	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.
M.8.5	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other (e.g., estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9; determine that the world population is more than 20 times larger).
M.8.6	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Cluster	**Understand the connections between proportional relationships, lines, and linear equations.**
M.8.7	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways (e.g., compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed).
M.8.8	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

Cluster	**Analyze and solve linear equations, pairs of simultaneous linear equations, and linear inequalities in one variable.**
M.8.9	Analyze and solve real-world and mathematical problems utilizing linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms.
M.8.10	Analyze and solve pairs of simultaneous linear equations by graphing, limiting to integer solutions. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
M.8.11	Explain each step in solving a linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
M.8.12	Analyze and solve real-world mathematical problems utilizing linear inequalities in one variable. Solve linear inequalities with rational number coefficients, including inequalities whose solutions require expanding expressions using the distributive property and combining like terms.
M.8.13	Rearrange formulas to isolate a given variable, using the same reasoning as in solving

	equations (e.g., rearrange Ohm's law V = IR to isolate resistance R). Instructional note: Limit to formulas with a linear focus.

Functions

Cluster	Define, evaluate, and compare functions.
M.8.14	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Instructional Note: Function notation is not required in grade 8.
M.8.15	Compare properties of two functions each represented in a different way, such as algebraically, graphically, numerically in tables, or by verbal descriptions (e.g., given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change).
M.8.16	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear (e.g., the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line).

Cluster	Use functions to model relationships between quantities.
M.8.17	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
M.8.18	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry

Cluster	Understand congruence and similarity using physical models, transparencies, or geometry software.
M.8.19	Verify experimentally the properties of rotations, reflections and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
M.8.20	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
M.8.21	Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.
M.8.22	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
M.8.23	Use informal arguments to establish facts about the angle sum and exterior angle of

	triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles (e.g., arrange three copies of the same triangle so that the sum of the three angles appears to form a line; give an argument in terms of transversals why this is so).

Cluster	**Understand and apply the Pythagorean Theorem.**
M.8.24	Explain a proof of the Pythagorean Theorem and its converse.
M.8.25	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
M.8.26	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Cluster	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**
M.8.27	Know the formulas for the volumes of cones, cylinders and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability

Cluster	**Investigate patterns of association in bivariate data.**
M.8.28	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association.
M.8.29	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.
M.8.30	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept (e.g., in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height).
M.8.31	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables (e.g., collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home; is there evidence that those who have a curfew also tend to have chores?).

High School Mathematics

At the high school level, the standards are organized by conceptual category (the number system, algebra, functions, geometry, modeling, and probability and statistics), showing the body of knowledge students should learn in each category to be college- and career-ready and to be prepared to study more advanced mathematics. There are two distinct course sequence pathways of the high school standards for the mathematics progression in grades 9-10.

- The Integrated Pathway is a course sequence of Mathematics I and Mathematics II each of which includes a blend of Algebra I and Geometry with data, probability, and statistics.
- The Traditional Pathway is a course sequence of Algebra I and Geometry with data, probability, and statistics included in each course.

Each pathway organizes the identical standards into courses that provide a strong foundation for post-secondary success. The content is simply grouped differently among the two years. See Appendix A: High School Mathematics I and Mathematics II Standards to see the organization of the Algebra I and Geometry standards in Mathematics I and Mathematics II. Local Education Agencies (LEA) must choose to implement either the Integrated or Traditional Pathway.

Regardless of the pathway chosen for grades 9-10, the third and fourth course options for all students are the same.

For LEAs interested in High School Algebra I for 8th Grade or High School Mathematics I for 8th Grade, see Appendix B.

TRADITIONAL PATHWAY

Mathematics – High School Algebra I

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in this course will focus on four critical domains that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Expressions and Equations	**Functions**
• Interpret algebraic expressions and transforming them purposefully to solve problems (e.g., in solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P).	• Understand contextual relationships of variables and constants (e.g., Annie is picking apples with her sister; the number of apples in her basket is described by n = 22t + 12, where t is the number of minutes Annie spends picking apples; what do the numbers 22 and 12 tell you about Annie's apple picking?).
Geometry	**Statistics and Probability**
• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances (e.g., find the area and perimeter of a real-world shape using a coordinate grid and Google Earth).	• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model (e.g., use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA).

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within High School Algebra I:

Expressions and Equations	
Interpret the structure of expressions and equations in terms of the context they model.	Standards 1-2
Extend the properties of exponents to rational exponents.	Standards 3-4

Write expressions in equivalent forms to solve problems.	Standard 5
Perform arithmetic operations on polynomials.	Standard 6
Create equations that describe numbers or relationships.	Standards 7-9
Solve equations and inequalities in one variable.	Standards 10-11
Solve systems of equations.	Standards 12-15
Represent and solve equations and inequalities graphically.	Standards 16-18
Functions	
Understand the concept of a function and use function notation.	Standards 19-21
Interpret functions that arise in applications in terms of a context.	Standard 22
Analyze functions using different representations.	Standards 23-25
Build a function that models a relationship between two quantities.	Standards 26-27
Build new functions from existing functions.	Standard 28
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 29
Geometry	
Use coordinates to prove simple geometric theorems algebraically.	Standards 30-31
Statistics and Probability	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 32-34
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standard 35
Interpret linear models.	Standards 36-37

Expressions and Equations

Cluster	**Interpret the structure of expressions and equations in terms of the context they model.**
M.A1HS.1	Interpret linear, exponential, and quadratic expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. c. Interpret the parameters in a linear function or exponential function of the form $f(x) = a*b^x$ in terms of a context.
M.A1HS.2	Use the structure of quadratic and exponential expressions to identify ways to rewrite them.

Cluster	Extend the properties of exponents to rational exponents.
M.A1HS.3	Explain the connections between expressions with rational exponents and expressions with radicals using properties of exponents. Extend from application of properties of exponents for expressions with integer exponents.
M.A1HS.4	Rewrite expressions involving radicals, including simplifying, and rational exponents using the properties of exponents.

Cluster	Write expressions in equivalent forms to solve problems.
M.A1HS.5	Choose and produce an equivalent form of linear, exponential, and quadratic expressions to reveal and explain properties of the quantity represented by the expression through connections to a graphical representation of the function. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression, when a=1 only, to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions in exponential functions. For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t}$ $\approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Cluster	Perform arithmetic operations on polynomials.
M.A1HS.6	Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Focus on linear or quadratic terms.

Cluster	Create equations that describe numbers or relationships.
M.A1HS.7	Create equations and inequalities in one variable, representing linear and exponential relationships, and use them to solve problems. In the case of exponential equations, limit to situations with integer inputs.
M.A1HS.8	Create equations in two or more variables, representing linear and exponential relationships between quantities. In the case of exponential equations, limit to situations with integer inputs.
M.A1HS.9	Represent constraints by linear equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

Cluster	Solve equations and inequalities in one variable.
M.A1HS.10	Solve linear equations including equations with coefficients represented by letters, simple exponential equations that rely on application of the laws of exponents, and compound linear inequalities in one variable.
M.A1HS.11	Solve quadratic equations in one variable by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square when a=1 only, and the quadratic formula, as appropriate for the initial form of the equation. a. Recognize the concept of complex solutions when the quadratic formula gives

	complex solutions. b. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$. Derive the quadratic formula from this method of completing the square.

Cluster	**Solve systems of equations.**
M.A1HS.12	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve simple cases by inspection (e.g., 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6). c. Solve real-word and mathematical problems leading to two linear equations in two variables (e.g., given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair).
M.A1HS.13	Understand and demonstrate ways to manipulate a system of two equations in two variables while preserving its solution set.
M.A1HS.14	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Include examples of solution sets with no solutions, an infinite number of solutions, and one solution.
M.A1HS.15	Solve a simple system consisting of a linear equation and a quadratic equation in two variables graphically.

Cluster	**Represent and solve equations and inequalities graphically.**
M.A1HS.16	Recognize that the graph of a linear or exponential equation in two variables is the set of all its solutions plotted in the coordinate plane.
M.A1HS.17	Explain why the x-coordinates of the points where the graphs of the linear and/or exponential equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations).
M.A1HS.18	Graph the solutions of a linear inequality in two variables as a half-plane and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions

Cluster	**Understand the concept of a function and use function notation.**
M.A1HS.19	Use multiple representations of linear and exponential functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically.
M.A1HS.20	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context.

M.A1HS.21	Recognize arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (e.g., the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n)+ f(n-1) for n ≥ 1).

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.A1HS.22	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its linear, exponential, and quadratic graphs and, where applicable, to the quantitative relationship it describes. a. Key features of linear and exponential graphs include: intercepts; and intervals where the function is increasing, decreasing, positive, or negative. b. Key features of quadratic graphs include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximum or minimum; symmetry; and end behavior.

Cluster	**Analyze functions using different representations.**
M.A1HS.23	Graph linear, exponential, and quadratic functions expressed symbolically and show key features of the graph. a. For linear functions, focus on intercepts. b. For exponential functions, focus on intercepts and end behavior. c. For quadratic functions, focus on intercepts, maxima, minima, end behavior, and the relationship between coefficients and roots to represent in factored form. Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.
M.A1HS.24	Compare properties of two linear, exponential, or quadratic functions each represented in a different way, such as algebraically, graphically, numerically in tables, or from verbal descriptions.
M.A1HS.25	Write a function defined by a linear, exponential, or quadratic expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square for a=1 only in a quadratic function to show zeros, extreme values, symmetry of the graph, the relationship between coefficients and roots represented in factored form and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions in exponential functions.

Cluster	**Build a function that models a relationship between two quantities.**
M.A1HS.26	Write linear, exponential, and quadratic functions that describe a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations.
M.A1HS.27	Construct linear and exponential functions, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship or given input-output pairs (include reading these from a table).

Cluster	Build new functions from existing functions.
M.A1HS.28	Identify the effect on the graphs of linear and exponential functions, f(x), with f(x) + k, and the graphs of quadratic functions, g(x), with g(x) + k, k g(x), g(kx), and g(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.A1HS.29	Distinguish between situations that can be modeled with linear functions, with exponential functions, and with quadratic functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. d. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. Extend the comparison of linear and exponential growth to quadratic growth.

Geometry

Cluster	Use coordinates to prove simple geometric theorems algebraically.
M.A1HS.30	Prove the slope criteria for parallel and perpendicular lines and use the slope criteria to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
M.A1HS.31	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. Instructional Note: Using the distance formula provides practice with the distance formula and its connection with the Pythagorean theorem.

Statistics and Probability

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.A1HS.32	Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots).
M.A1HS.33	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets.
M.A1HS.34	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.A1HS.35	Represent data on two quantitative variables on a scatter plot and describe how the variables are related.

	a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. Focus should be on situations for which linear models are appropriate. c. Fit a linear function for scatter plots that suggest a linear association.

Cluster	**Interpret linear models.**
M.A1HS.36	Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit.
M.A1HS.37	Distinguish between correlation and causation.

Mathematics - High School Geometry

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in this course will explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Basics of Geometry	**Transformations and Congruence**
• Prove theorems about triangles and other figures (e.g., that the sum of the measures of the angles in a triangle is 180^{o}). • Use coordinates and equations to describe geometric properties algebraically (e.g., write the equation for a circle in the plane with specified center and radius).	• Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.
Similarity and Trigonometry	**Circles**
• Apply knowledge of trigonometric ratios and the Pythagorean Theorem to determine distances in realistic situations (e.g., determine heights of inaccessible objects using various instruments, such as clinometers, hypsometers, transits, etc.).	• Apply theorems about circles to describe the relationships of components of a circle or formed by a circle and to find arc lengths and areas of sectors of circles.
Extending to Three Dimensions and Modeling	**Statistics and Probability**
• Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	• Work with probability and using ideas from probability in everyday situations (e.g., compare the chance that a person who smokes will develop lung cancer to the chance that a person who develops lung cancer smokes).

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within High School Geometry:

Basics of Geometry	
Experiment with transformations in the plane.	Standard 1
Identify and utilize inductive and deductive reasoning.	Standard 2

Prove geometric theorems.	Standard 3
Use coordinates to prove simple geometric theorems algebraically.	Standard 4
Make geometric constructions.	Standard 5
Transformations and Congruence	
Experiment with transformations in the plane.	Standards 6-9
Understand congruence in terms of rigid motions.	Standards 10-13
Prove geometric theorems.	Standards 14-15
Use coordinates to prove simple geometric theorems algebraically.	Standard 16
Similarity and Trigonometry	
Understand similarity in terms of similarity transformations.	Standards 17-19
Prove theorems involving similarity.	Standards 20-21
Define trigonometric ratios and solve problems involving right triangles.	Standards 22-24
Apply trigonometry to general triangles.	Standards 25-27
Circles	
Understand and apply theorems about circles.	Standards 28-29
Find arc lengths and areas of sectors of circles.	Standard 30
Make geometric constructions.	Standards 31-33
Extending to Three Dimensions and Modeling	
Explain volume formulas and use them to solve problems.	Standards 34-35
Visualize the relation between two-dimensional and three-dimensional objects and apply geometric concepts in modeling situations.	Standards 36-37
Statistics and Probability	
Understand independence and conditional probability and use them to interpret data.	Standards 38-42
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Standards 43-46
Use probability to evaluate outcomes of decisions.	Standards 47-48

Basics of Geometry

Cluster	**Experiment with transformations in the plane.**
M.GHS.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Cluster	**Identify and utilize inductive and deductive reasoning.**
M.GHS.2	Construct and justify the validity of a logical argument. a. Identify the converse, inverse, and contrapositive of a conditional statement.

	b. Translate a short, verbal argument into symbolic form. c. Use Venn diagrams to represent set relationships. d. Use inductive and deductive reasoning.

Cluster	Prove geometric theorems.
M.GHS.3	Use appropriate methods of proof to prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.

Cluster	Use coordinates to prove simple geometric theorems algebraically.
M.GHS.4	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Cluster	Make geometric constructions.
M.GHS.5	Make formal geometric constructions with a variety of tools and methods, such as a compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.: a. copying a segment; b. copying an angle; c. bisecting a segment; d. bisecting an angle; e. constructing perpendicular lines, including the perpendicular bisector of a line segment; and f. constructing a line parallel to a given line through a point not on the line.

Transformations and Congruence

Cluster	Experiment with transformations in the plane.
M.GHS.6	Build on prior knowledge from rigid motions to: a. represent transformations using geometric concepts in the plane. b. describe transformations as functions that take points in the plane as inputs and give other points as outputs. c. compare transformations that preserve distance and angle to those that do not.
M.GHS.7	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
M.GHS.8	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
M.GHS.9	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Describe a sequence of transformations that will carry a given figure onto another.

Cluster	Understand congruence in terms of rigid motions.
M.GHS.10	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

M.GHS.11	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
M.GHS.12	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
M.GHS.13	Use congruence criteria for triangles to solve problems and to prove relationships in geometric figures.

Cluster	**Prove geometric theorems.**
M.GHS.14	Use appropriate methods of proof to prove theorems about triangles and lines. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
M.GHS.15	Use appropriate methods of proof to prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.GHS.16	Use coordinates to prove simple geometric theorems about right triangles, quadrilaterals, and circles algebraically (e.g., derive the equation of a circle of given center and radius using the Pythagorean Theorem).

Similarity and Trigonometry

Cluster	**Understand similarity in terms of similarity transformations.**
M.GHS.17	Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
M.GHS.18	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
M.GHS.19	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Cluster	**Prove theorems involving similarity.**
M.GHS.20	Use appropriate methods of proof to prove theorems about triangles involving similarity. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

M.GHS.21	Use similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Use the Pythagorean Theorem and similarity criteria to derive and apply special right triangles to solve problems.

Cluster	**Define trigonometric ratios and solve problems involving right triangles.**
M.GHS.22	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
M.GHS.23	Explain and use the relationship between the sine and cosine of complementary angles.
M.GHS.24	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Cluster	**Apply trigonometry to general triangles.**
M.GHS.25	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.GHS.26	Prove the Laws of Sines and Cosines extending the definitions of sine and cosine to obtuse angles.
M.GHS.27	Understand and apply the Law of Sines and the Law of Cosines to solve problems and to find unknown measurements in right and non-right triangles.

Circles

Cluster	**Understand and apply theorems about circles.**
M.GHS.28	Prove that all circles are similar.
M.GHS.29	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Cluster	**Find arc lengths and areas of sectors of circles.**
M.GHS.30	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Cluster	**Make geometric constructions.**
M.GHS.31	Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.
M.GHS.32	Construct a tangent line from a point outside a given circle to the circle.
M.GHS.33	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Extending to Three Dimensions and Modeling

Cluster	**Explain volume formulas and use them to solve problems.**
M.GHS.34	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
M.GHS.35	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems, including how area and volume scale under similarity transformations.

Cluster	Visualize the relation between two-dimensional and three-dimensional objects and apply geometric concepts in modeling situations.
M.GHS.36	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
M.GHS.37	Use two- and three-dimensional shapes and circles, their measures, and their properties to describe objects. a. Apply concepts of density based on area and volume in modeling situations. b. Apply geometric methods to solve design problems to satisfy given constraints.

Statistics and Probability

Cluster	Understand independence and conditional probability and use them to interpret data.
M.GHS.38	Describe events as subsets of a sample space using characteristics of the outcomes or as unions, intersections, or complements of other events.
M.GHS.39	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities. Use this characterization to determine if they are independent.
M.GHS.40	Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
M.GHS.41	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
M.GHS.42	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Cluster	Use the rules of probability to compute probabilities of compound events in a uniform probability model.
M.GHS.43	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.
M.GHS.44	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model.
M.GHS.45	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B) and interpret the answer in terms of the model.
M.GHS.46	Use permutations and combinations to compute probabilities of compound events and solve problems.

Cluster	Use probability to evaluate outcomes of decisions.
M.GHS.47	Use probabilities to make fair decisions.
M.GHS.48	Analyze decisions and strategies using probability concepts.

THIRD AND FOURTH COURSE OPTIONS

Third and fourth personalized credit options are available to students on either pathway:

- High School Algebra II – Mathematics III
- High School Trigonometry/Pre-calculus – Mathematics IV
- Applied Statistics
- Transition Mathematics for Seniors (fourth course option only)
- Advanced Mathematical Modeling
- Calculus
- Financial Algebra/Mathematics
- Quantitative Reasoning
- Statistics
- Introduction to Mathematical Applications

The High School Mathematics course sequence may be prescribed at the county level. See Policy 2510 for a list of required mathematics courses.

Mathematics - High School Algebra II – Mathematics III

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in this course will build on their work with linear, quadratic, and exponential functions and extend their repertoire of functions to include polynomial, rational, and radical functions. (In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.) Students will work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. The Mathematical Habits of Mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

The Number System	**Expressions and Equations**
• Extend the number system to complex numbers and perform arithmetic operations with complex numbers.	• Derive the formula for the sum of a geometric series, and use the formula to solve problems (e.g., calculate mortgage payments).
Functions	**Statistics and Probability**
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision (e.g., estimate water and food needs in a disaster area, or use	• Make inferences and justify conclusions from sample surveys, experiments, and observational studies. • Analyze decisions and strategies using probability concepts (e.g., product testing,

volume formulas and graphs to find an optimal size for an industrial package).	medical testing, pulling a hockey goalie at the end of a game).

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within High School Algebra II – Mathematics III:

The Number System	
Perform arithmetic operations with complex numbers.	Standards 1-2
Expressions and Equations	
Use complex numbers in polynomial identities and equations.	Standards 3-5
Interpret the structure of expressions.	Standards 6-7
Write expressions in equivalent forms to solve problems.	Standard 8
Perform arithmetic operations on polynomials.	Standard 9
Understand the relationship between zeros and factors of polynomials.	Standards 10-11
Use polynomial identities to solve problems.	Standards 12-13
Rewrite rational expressions.	Standards 14-15
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 16
Represent and solve equations and inequalities graphically.	Standard 17
Solve systems of equations.	Standard 18
Functions	
Create equations that describe numbers or relationships.	Standards 19-21
Interpret functions that arise in applications in terms of a context.	Standards 22-23
Analyze functions using different representations.	Standards 24-26
Build a function that models a relationship between two quantities.	Standard 27
Build new functions from existing functions.	Standards 28-29
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 30
Statistics and Probability	
Summarize, represent, and interpret data on a single count or measurement variable.	Standard 31
Understand and evaluate random processes underlying statistical experiments.	Standards 32-33
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	Standards 34-37

Use probability to evaluate outcomes of decisions.	Standards 38-39

The Number System

Cluster	Perform arithmetic operations with complex numbers.
M.A2HS.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b representing real numbers.
M.A2HS.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Expressions and Equations

Cluster	Use complex numbers in polynomial identities and equations.
M.A2HS.3	Solve quadratic equations with real coefficients that have complex solutions.
M.A2HS.4	Factor special case polynomials with real coefficients that produce complex zeros.
M.A2HS.5	Show that the Fundamental Theorem of Algebra is true for quadratic polynomials with real coefficients.

Cluster	Interpret the structure of expressions.
M.A2HS.6	Interpret expressions including rational and polynomial expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
M.A2HS.7	Use the structure of expressions including polynomial and rational expressions to identify ways to rewrite them.

Cluster	Write expressions in equivalent forms to solve problems.
M.A2HS.8	Derive the formula for the sum of a finite geometric and use the formula to solve problems.

Cluster	Perform arithmetic operations on polynomials.
M.A2HS.9	Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Perform operations on polynomials with degree higher than two.

Cluster	Understand the relationship between zeros and factors of polynomials.
M.A2HS.10	Apply the Remainder Theorem to polynomial functions.
M.A2HS.11	Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

Cluster	Use polynomial identities to solve problems.
M.A2HS.12	Prove polynomial identities and use them to describe numerical relationships.

M.A2HS.13	Apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n with coefficients determined, for example, by Pascal's Triangle.

Cluster	**Rewrite rational expressions.**
M.A2HS.14	Rewrite simple rational expressions in different forms; write a(x)/b(x) in different forms using inspection, long division, synthetic division, or, for the more complicated examples, a computer algebra system.
M.A2HS.15	Recognize that rational expressions form a system analogous to the rational numbers, namely, they are closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
M.A2HS.16	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise

Cluster	**Represent and solve equations and inequalities graphically.**
M.A2HS.17	Explain why the x-coordinates of the points where the graphs of the linear, polynomial, rational, absolute value, exponential, and logarithmic equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations).

Functions

Cluster	**Create equations that describe numbers or relationships.**
M.A2HS.19	Create equations and inequalities in one variable, representing linear, quadratic, simple rational, and exponential relationships, and use them to solve problems.
M.A2HS.20	Create equations in two or more variables, representing linear, exponential, and quadratic relationships, between quantities.
M.A2HS.21	Represent constraints by linear, exponential, or quadratic equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.A2HS.22	Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its graph based on the behavior of data and context, and where applicable, to the quantitative relationship it describes. • Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; and end behavior.
M.A2HS.23	Select a model function based on behavior of data and context to calculate and interpret the average rate of change of linear, exponential, quadratic, and model functions based on behavior of data and context (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Cluster	Analyze functions using different representations.
M.A2HS.24	Graph quadratic, polynomial, square root, cube root, piecewise-defined functions, including step functions and absolute value functions, exponential, and logarithmic functions expressed symbolically and show key features of the graph. Use applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. a. For polynomial functions, focus on identifying zeros and showing end behavior. b. For exponential and logarithmic functions, focus on showing intercepts and end behavior. Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.
M.A2HS.25	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function focusing on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.
M.A2HS.26	Compare properties of two functions each represented in a different way, such as algebraically, graphically, numerically in tables, or by verbal descriptions. Focus on applications and how key features relate to characteristics of a situation.

Cluster	Build a function that models a relationship between two quantities.
M.A2HS.27	Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.

Cluster	Build new functions from existing functions.
M.A2HS.28	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Observe the effect of multiple transformations on a single graph and the common effect of each transformation across function types and use transformations to model situations.
M.A2HS.29	Find inverse functions for simple polynomial, simple rational, simple radical, and use simple exponential functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. Consider situations where the domain of the function must be restricted in order for the inverse to exist.

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.A2HS.30	For exponential models, express as a logarithm the solution to $a \bullet b^{ct} = d$, where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Statistics and Probability

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.A2HS.31	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Cluster	Understand and evaluate random processes underlying statistical experiments.
M.A2HS.32	Understand statistics as a process for making inferences about population parameters based on a random sample from that population. Compare theoretical and empirical results to evaluate the effectiveness.
M.A2HS.33	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Cluster	Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
M.A2HS.34	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
M.A2HS.35	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error using simulation models for random sampling. Informally develop the concepts of statistical significance and variability.
M.A2HS.36	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Recognize that some unlikely results can occur solely through randomness inherent in the system and "statistical significance" represents this likelihood. Make use of statistics as a way of dealing with, not eliminating, this inherent randomness.
M.A2HS.37	Evaluate reports based on data. Focus on data collection and how conclusions can be drawn from data.

Cluster	Use probability to evaluate outcomes of decisions.
M.A2HS.38	Use probabilities to make fair decisions, including situations involving quality control, false positive, and false negative results.
M.A2HS.39	Analyze decisions and strategies using probability concepts, including situations involving quality control, false positive, and false negative results.

Mathematics – High School Trigonometry/Pre-calculus – Mathematics IV

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in this course will generalize and abstract learning accumulated through previous courses as the final springboard to calculus. Students will take an extensive look at the relationships among complex numbers, vectors, and matrices. They will build on their understanding of functions, analyze rational functions using an intuitive approach to limits and synthesize functions by considering compositions and inverses. Students will expand their work with trigonometric functions and their inverses and complete the study of the conic sections begun in previous courses. They will enhance their understanding of probability by considering probability distributions and have previous experiences with series augmented. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Building Relationships among Complex Numbers, Vectors, and Matrices	**Analysis and Synthesis of Functions**
• Represent abstract situations involving vectors symbolically.	• Write a function that describes a relationship between two quantities (e.g., if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time).
Trigonometric and Inverse Trigonometric Functions of Real Numbers	**Derivations in Analytic Geometry**
• Make sense of the symmetry, periodicity, and special values of trigonometric functions using the unit circle. • Prove trigonometric identities and apply them problem solving situations.	• Make sense of the derivations of the equations of an ellipse and a hyperbola.
Series and Informal Limits	
• Apply mathematical induction to prove summation formulas.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within High School Trigonometry/Pre-calculus – Mathematics IV:

Building Relationships among Complex Numbers, Vectors, and Matrices	
Perform arithmetic operations with complex numbers.	Standard 1

Represent complex numbers and their operations on the complex plane.	Standards 2-4
Represent and model with vector quantities.	Standards 5-7
Perform operations on vectors.	Standards 8-9
Perform operations on matrices and use matrices in applications.	Standards 10-16
Solve systems of equations.	Standards 17-18
Trigonometric and Inverse Trigonometric Functions of Real Numbers	
Extend the domain of trigonometric functions using the unit circle.	Standards 19-22
Model periodic phenomena with trigonometric functions.	Standards 23-26
Prove and apply trigonometric identities.	Standards 27-28
Analysis and Synthesis of Functions	
Interpret functions that arise in applications in terms of a context.	Standard 29
Build a function that models a relationship between two quantities.	Standard 30
Analyze functions using different representations.	Standard 31
Build new functions from existing functions.	Standards 32-34
Derivations in Analytic Geometry	
Use conic sections to solve applications.	Standards 35-36
Series and Informal Limits	
Use sigma notations to evaluate finite sums.	Standards 37-38
Extend geometric series to infinite geometric series.	Standard 39

Building Relationships among Complex Numbers, Vectors, and Matrices

Cluster	**Perform arithmetic operations with complex numbers.**
M.4HSTP.1	Find the conjugate of a complex number; use conjugates to find moduli (magnitude) and quotients of complex numbers.

Cluster	**Represent complex numbers and their operations on the complex plane.**
M.4HSTP.2	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number.
M.4HSTP.3	Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
M.4HSTP.4	Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.

Cluster	**Represent and model with vector quantities.**
M.4HSTP.5	Recognize vector quantities as having both magnitude and direction. Represent vector

	quantities by directed line segments and use appropriate symbols for vectors and their magnitudes.
M.4HSTP.6	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
M.4HSTP.7	Solve problems involving velocity and other quantities that can be represented by vectors.

Cluster	**Perform operations on vectors.**
M.4HSTP.8	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Describe vector subtraction in terms of vector addition, represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
M.4HSTP.9	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. b. Compute the magnitude of a scalar multiple of a vector.

Cluster	**Perform operations on matrices and use matrices in applications.**
M.4HSTP.10	Use matrices to represent and manipulate data.
M.4HSTP.11	Multiply matrices by scalars to produce new matrices.
M.4HSTP.12	Add, subtract and multiply matrices of appropriate dimensions.
M.4HSTP.13	Demonstrate that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
M.4HSTP.14	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
M.4HSTP.15	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
M.4HSTP.16	Work with 2 × 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area.

Cluster	**Solve systems of equations.**
M.4HSTP.17	Represent a system of linear equations as a single matrix equation in a vector variable.
M.4HSTP.18	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Trigonometric and Inverse Trigonometric Functions of Real Numbers

Cluster	**Extend the domain of trigonometric functions using the unit circle.**
M.4HSTP.19	Understand radian measure of an angle as the length of the arc on the unit circle

	subtended by the angle.
M.4HSTP.20	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
M.4HSTP.21	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
M.4HSTP.22	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Cluster	**Model periodic phenomena with trigonometric functions.**
M.4HSTP.23	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
M.4HSTP.24	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
M.4HSTP.25	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context.
M.4HSTP.26	Solve multi-step trigonometric equations that require factoring or the use of identities.

Cluster	**Prove and apply trigonometric identities.**
M.4HSTP.27	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.
M.4HSTP.28	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Analysis and Synthesis of Functions

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.4HSTP.29	Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its graph based on the behavior of data and context, and, where applicable, to the quantitative relationship it describes. • Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity.

Cluster	**Build a function that models a relationship between two quantities.**
M.4HSTP.30	Write a function that describes a relationship between two quantities, including composition of functions.

Cluster	**Analyze functions using different representations.**
M.4HSTP.31	Graph trigonometric and rational functions expressed symbolically and show key features of the graph.

	a. For trigonometric functions, focus on period, midline, amplitude, and phase shift. b. For rational functions, focus on identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Analyze asymptotes and continuity informally using limits. Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.

Cluster	**Build new functions from existing functions.**
M.4HSTP.32	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Observe the effect of multiple transformations on a single graph and the common effect of each transformation across function types and use transformations to model situations.
M.4HSTP.33	Find inverse functions. a. Verify by composition that one function is the inverse of another. b. Read values of an inverse function from a graph or a table, given that the function has an inverse. Compute values of inverse functions from graphs and recognize the graph of an inverse function is the graph of the original function reflected about y=x. c. Produce an invertible function from a non-invertible function by restricting the domain.
M.4HSTP.34	Use an understanding of the inverse relationship between exponents and logarithmic functions to: a. Graph logarithms, b. Derive properties of logarithms, and c. Use these properties to model and solve problems and applications involving exponential and logarithmic functions.

Derivations in Analytic Geometry

Cluster	**Use conic sections to solve applications.**
M.4HSTP.35	Derive the equations of a parabola, circle, ellipse~~s~~, and hyperbola~~s~~ using their key components. Graph the equations of these conic sections. • Key components include asymptotes, center, directrix, foci, and radius.
M.4HSTP.36	Solve problems and applications that model conic sections.

Series and Informal Limits

Cluster	**Use sigma notations to evaluate finite sums.**
M.4HSTP.37	Develop sigma notation and use it to write series in equivalent form.
M.4HSTP.38	Apply the method of mathematical induction to prove summation formulas. For example, verify the sum of squares formula.

Cluster	Extend geometric series to infinite geometric series.
M.4HSTP.39	Develop intuitively that the sum of an infinite series of positive numbers can converge and derive the formula for the sum of an infinite geometric series. Apply infinite geometric series models intuitively.

Mathematics – Applied Statistics

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Applied Statistics provides authentic experiences in statistics designed to strengthen students' application of the statistical method. Students will conduct statistical simulations to model everyday situations in an increasingly data-rich world. Students in this course will select appropriate graphical and numerical methods to explore data, design and implement a plan to collect and analyze data, and use probability to evaluate outcomes and make decisions. Students will build on their work with linear, quadratic, and exponential functions and extend their repertoire of functions to include polynomial, radical, and rational functions. Students will use multiple representations, technology, applications and modeling in problem-solving contexts. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Exploring Data	**Designing Studies**
• Represent data visually and calculate statistical measures that describe the data set (e.g., construct and interpret a histogram for a student created data set).	• Design a plan to collect data using an appropriate sampling method to solve a problem (e.g., design and conduct an experiment to determine the effect of a treatment).
Functions and Modeling	**Probability and Informed Decisions**
• Explore expressions, functions, and models to highlight key features that provide insight into their structure, properties, and form (e.g., model situations using mathematics to provide an optimal solution).	• Make inferences and justify conclusions from data and analyze decisions and strategies using probability concepts (e.g., compare experimental and theoretical probabilities to make informed decisions).

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Applied Statistics:

Exploring Data	
Select appropriate graphical and numerical methods to explore data.	Standards 1-7
Designing Studies	
Design and implement a plan to collect and analyze data	Standards 8-12
Functions and Modeling	
Explore expressions, functions, and models to describe numbers or relationships.	Standards 13-21

Probability and Informed Decisions	
Use probability to evaluate outcomes and make decisions.	Standards 22-29

Exploring Data

Cluster	Select appropriate graphical and numerical methods to explore data.
M.ASHS.1	Generate appropriate ways to display various types of data. Instructional Note: Build on data displays introduced in prior courses.
M.ASHS.2	Calculate appropriate measures of center, variability, and position for data. Instructional note: Include comparisons of mean vs. median, standard deviation vs. IQR.
M.ASHS.3	Use graphical displays and summary statistics to make conclusions. Informally develop the concept of statistical significance; a result that is unlikely to have occurred by chance alone. Instructional Note: Focus on statistics as a way of dealing with, not eliminating, inherent randomness.
M.ASHS.4	Represent data in two variables to model relationships between quantities. Instructional Note: Students will use multiple representations with appropriate labels and scales.
M.ASHS.5	Select a function that models a relationship between two quantities and interpret key features of graphs and tables in terms of the quantities. Instructional Note: Focus on form, strength, direction, and departures from a model based on data and context.
M.ASHS.6	Compare characteristics of two data sets each represented in different ways (algebraically, graphically, numerically, and verbally). Instructional Note: Focus on applications and how key features relate to characteristics of a situation and select an appropriate model.
M.ASHS.7	Use appropriate measures of center and spread to describe a distribution. Instructional Note: Emphasize that only some data are well described by a normal distribution.

Designing Studies

Cluster	Design and implement a plan to collect and analyze data.
M.ASHS.8	Develop a process for making inferences about population parameters based on a random sample through data collection and analysis.
M.ASHS.9	Evaluate the results from a given data-generating process to determine consistency between theoretical and experimental probabilities. Instructional Note: Include the Law of Large Numbers.
M.ASHS.10	Recognize the purposes of and differences among sample surveys, experiments, and observational studies. Explain the importance of randomization in each method. Instructional Note: Emphasize that the way in which data is collected determines the scope and nature of the conclusions.
M.ASHS.11	Use data from a sample survey to estimate a population mean or proportion. Instructional Note: Develop the connection between sample size and margin of error.
M.ASHS.12	Design and conduct an experiment to compare two treatments. Instructional Note: Include randomization, replication, blocking, and control in the design.

Functions and Modeling

Cluster	Explore expressions, functions, and models to describe numbers or relationships.
M.ASHS.13	Create equations and inequalities in one variable, representing linear, exponential, quadratic, and simple rational functions, and use them to solve problems.
M.ASHS.14	Develop the concept of a complex number i such that $i^2 = -1$. Understand that every complex number can be written in the form a + bi with a and b real.
M.ASHS.15	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
M.ASHS.16	Use the structure of polynomial and rational expressions to identify ways to rewrite them.
M.ASHS.17	Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
M.ASHS.18	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. Instructional Note: This standard requires the general division algorithm for polynomials.
M.ASHS.19	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.
M.ASHS.20	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations (e.g., solve $z = \frac{x-\mu}{\sigma}$ for σ and Margin of Error = $z * \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ for n). Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. This example applies to earlier instances of this standard, not to the current course.
M.ASHS.21	Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. • Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative extrema; symmetries; and end behavior. Instructional Note: Emphasize the selection of a model function based on the behavior of data in context.

Cluster	Use probability to evaluate outcomes and make decisions.
M.ASHS.22	Connect sampling variability and margin of error to generate and interpret plausible parameter values. Instructional Note: The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred by chance alone. Focus on statistics as a way of dealing with, not eliminating, inherent randomness.
M.ASHS.23	Interpret results from a randomized experiment comparing two treatments. Use simulations to decide if experimental results are significant. Instructional Note:

	Develop informally the comparison of an observed result and an established probability value (for example $p \leq 0.05$).
M.ASHS.24	Evaluate claims based on data reports. Instructional Note: Data reports can be gathered from media.
M.ASHS.25	Use probability rules to make fair decisions. Instructional Note: Extend and apply probability rules introduced in prior courses to more complex probability models that involve decisions. Include examples that yield both false positive and false negative results.
M.ASHS.26	Use two-way tables, tree diagrams, Venn diagrams, or 10 x 10 grids to model probabilities.
M.ASHS.27	Justify a decision using probability rules (e.g., product testing, medical testing, weather forecasting, marketing, or sports coaching decisions). Instructional Note: Extend and apply probability rules introduced in prior courses to more complex probability models that involve decisions. Include examples that yield both false positive and false negative results.
M.ASHS.28	Perform appropriate calculations for given outcomes and decisions based on expected values for non-normal distributions. Instructional Note: Focus on uniform, discrete, continuous (geometric areas), or games of chance.
M.ASHS.29	Given data from a normal distribution, use the mean and standard deviation to estimate population percentages. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Recognize that there are data sets for which such a procedure is not appropriate. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities).

Mathematics – Transition Mathematics for Seniors

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Transition Mathematics for Seniors prepares students for their entry-level, credit-bearing liberal studies mathematics course at the post-secondary level. Students will solidify their quantitative literacy by enhancing numeracy and problem-solving skills as they investigate and use the fundamental concepts of algebra, geometry, and introductory trigonometry. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Number and Quantity: The Real Number System The Complex Number System	**Algebra:** Seeing Structure in Expressions Arithmetic with Polynomials and Rational Expressions Creating Equations Reasoning with Equations and Inequalities
• Develop an understanding of basic operations, equivalent representations, and properties of the real and complex number systems.	• Create equations or inequalities that model physical situations. • Solve systems of equations, with an emphasis on efficiency of solution as well as reasonableness of answers, given physical limitations.
Functions: Interpreting Functions Building Functions	**Geometry:** Geometric Measuring and Dimension Expressing Geometric Properties with Equations Modeling with Geometry
• Develop knowledge and understanding of the concept of functions as they use, analyze, represent and interpret functions and their applications.	• Use coordinates and to prove geometric properties algebraically.
Statistics and Probability: Interpreting Categorical and Quantitative Data Making Inferences and Justifying Conclusions	
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Transition Mathematics for Seniors:

Number and Quantity – The Real Number System	
Reason quantitatively and use units to solve problems.	Standards 1-2
Number and Quantity – The Complex Number System	
Use complex numbers in polynomial identities and equations.	Standard 3
Algebra – Seeing Structure in Expressions	
Interpret the structure of expressions.	Standard 4
Write expressions in equivalent forms to solve problems.	Standard 5
Understand the connections between proportional relationship, lines, and linear equations.	Standards 6-8
Algebra – Arithmetic with Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials.	Standard 9
Algebra – Creating Equations	
Create equations that describe numbers or relationships.	Standards 10-13
Algebra – Reasoning with Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 14
Solve equations and inequalities in one variable.	Standards 15-17
Solve systems of equations.	Standards 18-20
Represent and solve equations and inequalities graphically.	Standards 21-22
Functions – Interpreting Functions	
Understand the concept of a function and use function notation.	Standard 23
Interpret functions that arise in applications in terms of the context.	Standards 24-27
Analyze functions using different representations.	Standards 28-34
Functions – Building Functions	
Build a function that models a relationship between two quantities.	Standards 35-36
Geometry – Geometric Measuring and Dimension	
Explain volume formulas and use them to solve problems.	Standards 37-38
Geometry – Expressing Geometric Properties with Equations	
Use coordinates to prove simple geometric theorems algebraically.	Standards 39-40
Geometry – Modeling with Geometry	
Apply geometric concepts in modeling situations.	Standard 41
Statistics and Probability – Interpreting Categorical and Quantitative Data	
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 42-45
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 46-49

Statistics and Probability – Making Inferences and Justifying Conclusions	
Understand and evaluate random processes underlying statistical experiments.	Standard 50

Number and Quantity – The Real Number System

Cluster	Reason quantitatively and use units to solve problems.
M.TMS.1	Use units to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.TMS.2	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Number and Quantity – The Complex Number System

Cluster	Use complex numbers in polynomial identities and equations.
M.TMS.3	Solve quadratic equations with real coefficients that have complex solutions.

Algebra – Seeing Structure in Expressions

Cluster	Interpret the structure of expressions.
M.TMS.4	Use the structure of quadratic and exponential expressions to identify ways to rewrite them.

Cluster	Write expressions in equivalent forms to solve problems.
M.TMS.5	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Cluster	Understand the connections between proportional relationship, lines, and linear equations.
M.TMS.6	Graph proportional relationships, interpreting the unit rates as the slope of the graph. Compare two different proportional relationships represented in different ways.
M.TMS.7	Explain (e.g., using similar triangles) why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.
M.TMS.8	Solve linear equations in one variable.

Algebra – Arithmetic with Polynomials and Rational Expressions

Cluster	Perform arithmetic operations on polynomials.
M.TMS.9	Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials.

Algebra – Creating Equations

Cluster	Create equations that describe numbers or relationships.
M.TMS.10	Create equations and inequalities in one variable, representing linear, exponential, quadratic, and simple rational relationships, and use them to solve problems.
M.TMS.11	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
M.TMS.12	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
M.TMS.13	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Algebra – Reasoning with Equations and Inequalities

Cluster	Understand solving equations as a process of reasoning and explain the reasoning.
M.TMS.14	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

Cluster	Solve equations and inequalities in one variable.
M.TMS.15	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
M.TMS.16	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
M.TMS.17	Solve quadratic equations in one variable by inspection (e.g., $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Recognize the concept of complex solutions when the quadratic formula gives complex solutions, and write them as a ± bi for real numbers a and b.

Cluster	Solve systems of equations.
M.TMS.18	Understand and demonstrate ways to manipulate a system of two equations in two variables while preserving its solution set.
M.TMS.19	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Instructional Note: Include examples of solution sets with no solutions, an infinite number of solutions, and one solution.
M.TMS.20	Explain why the x-coordinates of the points where the graphs of the linear, polynomial, rational, absolute value, and exponential equations y = f(x) and y = g(x) intersect are the solution of the equation f(x) = g (x); find the solution approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations).

Cluster	Represent and solve equations and inequalities graphically.
M.TMS.21	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Include examples of solution sets with no solutions, an infinite number of solutions, and one solution.
M.TMS.22	Graph the solutions to a linear inequality in two variables as a half-plane graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions – Interpreting Functions

Cluster	Understand the concept of a function and use function notation.
M.TMS.23	Use multiple representations of linear and exponential functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically.

Cluster	Interpret functions that arise in applications in terms of the context.
M.TMS.24	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
M.TMS.25	Interpret the parameters in a linear or exponential function in terms of a context.
M.TMS.26	Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. • Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity.
M.TMS.27	Distinguish between situations that can be modeled with linear functions and with exponential functions.

Cluster	Analyze functions using different representations.
M.TMS.28	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
M.TMS.29	Describe qualitatively the functional relationship between two quantities by analyzing a graph.
M.TMS.30	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.
M.TMS.31	Graph linear, quadratic, and polynomial functions expressed symbolically and show key features of the graph. a. For linear functions, focus intercepts. b. For quadratic functions, focus on intercepts, maxima, minima, end behavior, and the relationship between coefficients and roots to represent in factored form. c. For polynomial functions, focus on identifying zeros and showing end behavior. Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.

M.TMS.32	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasingly linearly, quadratically, or (more generally) as a polynomial function.
M.TMS.33	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
M.TMS.34	Compare properties of two functions each represented in a different way, such as algebraically, graphically, numerically in tables, or by verbal descriptions.

Functions - Building Functions

Cluster	**Build a function that models a relationship between two quantities.**
M.TMS.35	Construct linear and exponential functions, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship, or given input-output pairs (include reading these from a table).
M.TMS.36	Write a function that describes a relationship between two quantities. a. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. b. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Geometry – Geometric Measuring and Dimension

Cluster	**Explain volume formulas and use them to solve problems.**
M.TMS.37	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
M.TMS.38	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Geometry – Expressing Geometric Properties with Equations

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.TMS.39	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
M.TMS.40	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. Instructional Note: Using the distance formula provides practice with the distance formula and its connection with the Pythagorean theorem.

Geometry – Modeling with Geometry

Cluster	Apply geometric concepts in modeling situations.
M.TMS.41	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios).

Statistics and Probability - Interpreting Categorical & Quantitative Data

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.TMS.42	Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Interpret linear models.
M.TMS.43	Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit.
M.TMS.44	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
M.TMS.45	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.TMS.46	Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots).
M.TMS.47	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets.
M.TMS.48	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
M.TMS.49	Distinguish between correlation and causation.

Statistics and Probability - Interpreting Categorical & Quantitative Data

Cluster	Understand and evaluate random processes underlying statistical experiments.
M.TMS.50	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Mathematics – Advanced Mathematical Modeling

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Primary focal points of Advanced Mathematical Modeling include the analysis of information using statistical methods and probability, modeling change and mathematical relationships, mathematical decision making in finance, and spatial and geometric modeling for decision-making. Students will learn to become critical consumers of the quantitative data that surround them every day, knowledgeable decision makers who use logical reasoning and mathematical thinkers who can use their quantitative skills to solve problems related to a wide range of situations. As students solve problems in various applied situations, they will develop critical skills for success in college and careers, including investigation, research, collaboration and both written and oral communication of their work. As students work with these topics, they will rely on mathematical processes, including problem-solving techniques, appropriate mathematical language and communication skills, connections within and outside mathematics and reasoning. Students will use applications and modeling, multiple representations, technology, and numerical fluency in problem-solving contexts. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Developing College and Career Skills	**Finance**
• Develop and apply skills used in college and careers, including reasoning, planning and communication, to make decisions and solve problems in applied situations.	• Create and analyze mathematical models to make decisions related to earning, investing, spending and borrowing money.
Probability	**Statistics**
• Use basic rules of counting and probability to analyze and evaluate risk and return in the context of everyday situations.	• Make decisions based on understanding, analysis and critique of reported statistical information and summaries.
Modeling	**Networks**
• Analyze numerical data in everyday situations using a variety of quantitative measures and numerical processes.	• Use a variety of network models represented graphically to organize data in quantitative situations, make informed decisions, and solve problems.
Social Decision Making	**Geometry**
• Analyze the mathematics behind various methods of ranking and selection and consider the advantages/disadvantages of each method.	• Solve geometric problems involving inaccessible distances. • Use vectors to solve applied problems.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Advanced Mathematical Modeling:

Developing College and Career Skills	
Mathematics as a language.	Standards 1-2
Tools for problem solving.	Standard 3
Finance	
Understand financial models.	Standards 4-6
Personal use of finance.	Standards 7-8
Probability	
Analyze information using probability and counting.	Standards 9-10
Manage uncertainty.	Standards 11-12
Statistics	
Critique statistics.	Standards 13-16
Perform statistical analysis.	Standards 17-21
Communicate statistical information.	Standards 22-23
Modeling	
Manage numerical data.	Standards 24-25
Model data and change with functions.	Standards 26-30
Networks	
Network for decision making.	Standards 31-32
Social Decision Making	
Make decisions using ranking and voting.	Standards 33-34
Geometry	
Concrete geometric representation (physical modeling).	Standards 35-36
Abstract geometric representation (matrix modeling).	Standards 37-38

Developing College and Career Skills

Cluster	Mathematics as a language.
M.AMM.1	Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical arguments and analyze the soundness of mathematical arguments of others.
M.AMM.2	Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.

Cluster	Tools for problem solving.
M.AMM.3	Gather data, conduct investigations, and apply mathematical concepts and models to solve problems in mathematics and other disciplines.

Finance

Cluster	Understand financial models.
M.AMM.4	Determine, represent, and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., business loans, auto,

	mortgage, and/or credit card).
M.AMM.5	Determine, represent, and analyze mathematical models for investments involving simple and compound interest with and without additional deposits (e.g., savings accounts, bonds, and/or certificates of deposit).
M.AMM.6	Determine, represent, and analyze mathematical models for inflation and the Consumer Price Index using concepts of rate of change and percentage growth.

Cluster	**Personal use of finance.**
M.AMM.7	Research and analyze personal budgets based on given parameters (e.g., fixed and discretionary expenses, insurance, gross vs. net pay, types of income, wage, salary, commission, career choice, geographic region, retirement and/or investment planning).
M.AMM.8	Research and analyze taxes including payroll, sales, personal property, real estate, and income tax returns.

Probability

Cluster	**Analyze information using probability and counting.**
M.AMM.9	Use the Fundamental Counting Principle, permutations, and combinations to determine all possible outcomes for an event; determine probability and odds of a simple event; explain the significance of the Law of Large Numbers.
M.AMM.10	Determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, two-way frequency tables and area models, to make decisions in problem situations.

Cluster	**Manage uncertainty.**
M.AMM.11	Use probabilities to make and justify decisions about risks in everyday life.
M.AMM.12	Calculate expected value to analyze mathematical fairness, payoff and risk.

Statistics

Cluster	**Critique-statistics.**
M.AMM.13	Identify limitations or lack of information in studies reporting statistical information, especially when studies are reported in condensed form.
M.AMM.14	Interpret and compare the results of polls, given a margin of error.
M.AMM.15	Identify uses and misuses of statistical analyses in studies reporting statistics or using statistics to justify particular conclusions, including assertions of cause and effect versus correlation.
M.AMM.16	Describe strengths and weaknesses of sampling techniques, data and graphical displays and interpretations of summary statistics, and other results appearing in a study, including reports published in the media.

Cluster	**Perform statistical analysis.**
M.AMM.17	Identify the population of interest, select an appropriate sampling technique, and collect data.
M.AMM.18	Identify the variables to be used in a study.

M.AMM.19	Determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results.
M.AMM.20	Create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features.
M.AMM.21	Determine possible sources of variability of data, both those that can be controlled and those that cannot be controlled.

Cluster	**Communicate statistical information.**
M.AMM.22	Report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied.
M.AMM.23	Communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language.

Modeling

Cluster	**Manage numerical data.**
M.AMM.24	Solve problems involving large quantities that are not easily measured.
M.AMM.25	Use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems.

Cluster	**Model data and change with functions.**
M.AMM.26	Determine or analyze an appropriate model for problem situations - including linear, quadratic, power, exponential, logarithmic and logistic functions (e.g., stopping distance, period of a pendulum, population growth, Richter Scale, and/or Fujita Tornado Scale).
M.AMM.27	Determine or analyze an appropriate cyclical model for problem situations that can be modeled with trigonometric functions (e.g., predator-prey models, tide heights, diurnal cycle, and/or music).
M.AMM.28	Determine or analyze an appropriate piecewise model for problem situations (e.g., postal rates, phase change graphs, sales tax, and/or utility usage rates).
M.AMM.29	Solve problems using recursion or iteration (e.g., fractals, compound interest, population growth or decline, and/or radioactive decay).
M.AMM.30	Collect numerical bivariate data; use the data to create a scatter plot; determine whether or not a relationship exists; if so, select a function to model the data, justify the selection and use the model to make predictions.

Networks

Cluster	**Network for decision making.**
M.AMM.31	Solve problems involving scheduling or routing situations that can be represented by a vertex-edge graph; find critical paths, Euler paths, Hamiltonian paths, and minimal spanning trees (e.g., Konigsberg bridge problem, mail vs. Fed Ex delivery routes, Kolam drawings of India, traveling salesman problem, and/or map coloring).
M.AMM.32	Construct, analyze, and interpret flow charts in order to develop and describe problem solving procedures.

Social Decision Making

Cluster	Make decisions using ranking and voting.
M.AMM.33	Apply and analyze various ranking algorithms to determine an appropriate method for a given situation (e.g., fair division, apportionment, and/or search engine results).
M.AMM.34	Analyze various voting and selection processes to determine an appropriate method for a given situation (e.g., preferential vs. non-preferential methods, and/or weighted voting).

Geometry

Cluster	Concrete geometric representation (physical modeling).
M.AMM.35	Create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications.
M.AMM.36	Solve geometric problems involving inaccessible distances.

Cluster	Abstract geometric representation (matrix modeling).
M.AMM.37	Use vectors to represent and solve applied problems.
M.AMM.38	Use matrices to represent geometric transformations and solve applied problems.

Mathematics - Calculus

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students will deepen and extend their understanding of functions, continuity, limits, differentiation, applications of derivatives, integrals, and applications of integration. Students will apply the Rule of Four (Numerical, Analytical, Graphical and Verbal) throughout the course and use available technology to enhance learning. Student will use graphing utilities to investigate concepts and to evaluate derivatives and integrals. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Algebra

- A utility company burns coal to generate electricity. The cost C in dollars of removing p% of the air pollutants emissions is $C = \frac{90,000p}{100-p}, \quad 0 \le p < 100.$ Find the cost of removing (a) 10%, (b) 25%, and (c) 75% of the pollutants. Find the limit of C as $p \to 100^{-}$.
- A management company is planning to build a new apartment complex. Knowing the maximum number of apartments that the lot can hold and given a function for the maintenance costs, determine the number of apartments that will minimize the maintenance costs.
- The velocity v of the flow of blood at a distance r from the central axis of an artery of radius R is $v = k\left(R^2 - r^2\right)$ where k is the constant of proportionality. Find the average rate of flow of blood along a radius of the artery (use 0 and R as the limits of integration).

Data Analysis and Probability

- The average data entry speeds S (words per minute) of a business student after t weeks of lessons are recorded in the following table.

t	5	10	15	20	25	30
S	28	56	79	90	93	94

Geometry

- The radius of a right circular cylindrical balloon is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}\sqrt{t}$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time.
- Given 50 meters of framing material, construct a window that will let in the most light if the middle of the window is a rectangle and the top and bottom of the window are semi-circles.
- The graph of f consists of the three line segments joining the points (0,0), (2, -2), (6, 2), and (8,3). The function F is defined as follows $F(x) = \int_0^x f(t)\,dt.$ Find the total enclosed areas generated by f and the x-axis. Determine the points of inflection of F on the interval $(0,8)$.

A model for the data is $S = \frac{100t^2}{65+t}, \quad t > 0.$ Do you think that there is a limiting speed? If so, what is the limiting speed? If not, why? • Identify a real-world situation that involves quantities that change over time and develop a method to collect and analyze related data. Develop a continuous function to model the data and generalize the results to make a conclusion. • A sheet of typing paper is ruled with parallel lines that are 2 inches apart. A two-inch needle is tossed randomly onto the sheet of paper. The probability that the needle will touch a line is $P = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin\theta \, d\theta$ where θ is the acute angle between the needle and any one of the parallel lines. Find the probability.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Calculus:

Algebra	
Understand the key concepts, connections, and applications of functions, limits, continuity, derivatives, and integrals represented in multiple ways.	Standards 1-19
Geometry	
Apply the key concepts, connections, and applications of limits, continuity, derivatives, and integration for a wide variety of regions.	Standards 20-22
Data Analysis and Probability	
Apply the key concepts and applications of limits, continuity, derivatives, and integration to analyze functions that represent a collection of data.	Standard 23

Algebra

Cluster	**Understand the key concepts, connections and applications of functions, limits, continuity, derivatives, and integrals represented in multiple ways.**
M.C.1	Use abstract notation to apply properties of algebraic, trigonometric, exponential, logarithmic, and composite functions, as well as their inverses, represented graphically,

	numerically, analytically, and verbally; and demonstrate an understanding of the connections among these representations.
M.C.2	Demonstrate a conceptual understanding of the definition of a limit via the analysis of continuous and discontinuous functions represented using multiple representations (e.g., graphs and tables).
M.C.3	Use the properties of limits including addition, product, quotient, composition, and squeeze/sandwich theorem to calculate the various forms of limits: one-sided limits, limits at infinity, infinite limits, limits that do not exist, and special limits such as $\lim_{x\to 0}\frac{\sin x}{x}=1$ and $\lim_{x\to 0}\frac{1-\cos x}{x}=0$.
M.C.4	Apply the definition of continuity to determine where a function is continuous or discontinuous including continuity at a point, continuity over an interval, application of the Intermediate Value Theorem, and graphical interpretation of continuity and discontinuity.
M.C.5	Investigate and apply the definition of the derivative graphically, numerically, and analytically at a point; conceptually interpret the derivative as an instantaneous rate of change and the slope of the tangent line.
M.C.6	Discriminate between the average rate of change and the instantaneous rate of change using real-world problems.
M.C.7	Recognize when the Extreme Value Theorem indicates that function extrema exist.
M.C.8	Quickly recall and apply rules of differentiation including the constant multiple rule, the sum rule, the difference rule, the product rule, the quotient rule, the power rule, and the chain rule as applied to algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions using techniques of both explicit and implicit differentiation.
M.C.9	Apply Rolle's Theorem and the Mean Value Theorem to real-world problems.
M.C.10	Construct and use mathematical models to solve optimization, related-rates, velocity, and acceleration problems.
M.C.11	Determine antiderivatives that follow from derivatives of basic functions and apply substitution of variables.
M.C.12	Evaluate definite integrals using basic integration properties such as addition, subtraction, constant multipliers, the power rule, substitution, and change of limits.
M.C.13	Characterize the definite integral as the total change of a function over an interval and use this to solve real-world problems.
M.C.14	Apply the Fundamental Theorem of Calculus to evaluate definite integrals and to formulate a cumulative area function and interpret the function as it relates to the integrand.
M.C.15	Use limits to deduce asymptotic behavior of the graph of a function.
M.C.16	Compare and contrast the limit definition (not delta epsilon) of continuity and the graphical interpretation of the continuity of a function at a point; recognize different types of discontinuities.
M.C.17	Develop tangent lines as best linear approximations to functions near specific points; explain this conceptually; construct these tangent lines; and apply this concept to Newton's Method.
M.C.18	Investigate and explain the relationships among the graphs of a function, its derivative and its second derivative; construct the graph of a function using the first and second derivatives including extrema, points of inflection, and asymptotic behavior.

M.C.19	Approximate areas under a curve using Riemann sums by applying and comparing left, right, and midpoint methods for a finite number of subintervals.

Geometry

Cluster	**Apply the key concepts, connections and applications of limits, continuity, derivatives, and integration for a wide variety of regions.**
M.C.20	Justify why differentiability implies continuity and classify functional cases when continuity does not imply differentiability.
M.C.21	Calculate a definite integral using Riemann sums by evaluating an infinite limit of a sum using summation notation and rules for summation.
M.C.22	Use integration to solve problems that involve linear displacement, total distance, position, velocity, acceleration, and area between curves by looking at both functions of x and functions of y; utilize units to interpret the physical nature of the calculus process.

Data Analysis and Probability

Cluster	**Apply the key concepts and applications of limits, continuity, derivatives, and integration to analyze functions that represent a collection of data.**
M.C.23	Identify a real-world situation that involves quantities that change over time; pose a question; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize, and analyze related data; extend the nature of collected, discrete data to that of a continuous function that describes the known data set; generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically, and verbally using the predictive and analytic tools of calculus.

Mathematics – Financial Algebra/Mathematics

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in this course will focus on financial applications designed to deepen and extend understanding of mathematics. Students in Financial Algebra/Mathematics will communicate effectively, using accurate mathematical language in a financial context. Students will interpret and analyze various functions, graphs and data in order to make responsible and wise financial decisions in the context of their personal lives regarding banking services, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Mathematical Language in a Financial Context	**The Algebra of Finance**
• Demonstrate reasoning skills in developing, explaining, and justifying sound financial decisions. • Communicate effectively, using accurate mathematical language in a financial context.	• Apply algebraic skills and concepts to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions.
Financial Modeling with Functions	**Modeling with Data**
• Interpret and analyze various functions, graphs, and data to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions.	• Create, interpret, and evaluate financial models to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Financial Algebra/Mathematics:

Mathematical Language in a Financial Context	
Communicate reasoning and decisions.	Standards 1-3
Algebra/Mathematics of Finance	
Use algebraic reasoning and techniques.	Standards 4-11
Financial Modeling with Functions	
Construct, graph, use, and interpret functions.	Standards 12-27
Financial Modeling with Data	
Represent, summarize, and evaluate data.	Standards 28-37

Mathematical Language in a Financial Context

Cluster	Communicate reasoning and decisions.
M.FAM.1	Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical decision making (e.g., demonstrate reasoning skills in creating and presenting a budget of monthly expenses based on a career pathway income, and analyze the soundness of the mathematical reasoning of others; determine outlook for a chosen career pathway and use the average salary to determine if the desired cost of living can be met).
M.FAM.2	Communicate with and about mathematics in a financial context.
M.FAM.3	Communicate with and about mathematics in writing and orally, both independently and collaboratively, by preparing financial plans (e.g., plan for an emergency savings fund that will last three to six months in the case of loss of income; determine the total percentage of income paid to taxes or the percentage of total salary that a benefits package represents).

Algebra/Mathematics of Finance

Cluster	Use algebraic reasoning and techniques.
M.FAM.4	Interpret parts of an expression or equation, such as terms, factors, and coefficients, in a variety of financial models including those found in stock markets, automobile financing, and in banking contexts.
M.FAM.5	Create and solve linear equations and inequalities in one variable and use them to solve problems in financial applications that may include, but are not limited to, stock markets, automobile ownership, business modeling, or employment (e.g., calculate wages by hourly rates or pay periods to make decisions about pay in a real-world context).
M.FAM.6	Create equations in two or more variables to represent relationships between quantities in a financial context; graph equations on coordinate axes with labels and scales. Financial contexts may include, but are not limited to, stock markets, automobile ownership, business modeling employment, banking, consumer debt, and independent living decisions regarding taxes or planning for retirement (e.g., create a linear expense equation based on fixed and variable expenses and graph choosing an appropriate scale and origin for the graph).
M.FAM.7	Represent constraints in financial applications by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context (e.g., create a system of equations based on the expenses incurred and monthly payment when choosing home ownership versus rental; find the percentage of total salary that a benefits package represents; calculate taxes owed based on a given income and tax table and determine total percentage of income paid to taxes; calculate the gross pay and net pay using the FICA percentage (7.65%), retirement contribution, and worker's compensation insurance (employer match)).
M.FAM.8	Rearrange formulas for financial applications to highlight a quantity of interest, using the same reasoning as in solving equations. Know difference between growth and decay functions (e.g., solve the literal equation for exponential depreciation to find a depreciation rate and the literal equation for continuous interest to find the interest

	rate; apply the formula for average daily balance, (average daily balance*APR*days in billing cycle)/365, using literal equations with varying APRs and billing cycles).
M.FAM.9	Solve systems of linear equations exactly and approximately (with graphs) in making financial decisions, focusing on pairs of linear equations in two variables (e.g., create and solve a system of equations based on the expenses incurred and monthly payment when choosing home ownership versus rental).
M.FAM.10	Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials (e.g., combine the polynomials that model income and expense to create a profit model).
M.FAM.11	Solve quadratic equations in one variable in a financial context that may include, but are not limited to, business modeling or employment decisions (e.g., given a quadratic equation that models a profit function, determine the break-even points; apply braking distance/stopping distance formulas to solve problems related to driving and safety data).

Financial Modeling with Functions

Cluster	**Construct, graph, use, and interpret functions.**
M.FAM.12	Use functions to model financial situations. Use multiple representations of functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically (e.g., develop and communicate the appropriateness of representing a commission salary using a linear versus a piecewise function; use linear and polynomial functions to evaluate and communicate quantities as required by Internal Revenue Service and Social Security Administration regulations and to determine when and why the models may be discontinuous).
M.FAM.13	Use function notation in financial applications, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a financial context (e.g., in making decisions regarding retirement income, apply the formula $A(t) = Pe^{rt}$ to determine future value).
M.FAM.14	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. Use this relationship in analyzing financial situations (e.g., compare the linear function modeling simple interest with the exponential function modeling compound interest).
M.FAM.15	Select a function that models a relationship between two quantities in financial contexts, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship (e.g., write, graph, and interpret the revenue (quadratic) function in comparison to the expense (linear) function using key features of the functions; reason quantitatively to compare subsidized and unsubsidized loans, as well as other forms of financial aid available to college students; calculate mortgage payments, reasoning and making decisions about the length of the loan and a fixed versus adjustable rate mortgage).
M.FAM.16	Interpret the parameters in a linear or exponential function in terms of a context (e.g., investigate and compare, using technology and regression, historical data to determine if automobile depreciation follows a linear or exponential model).

M.FAM.17	Construct linear and exponential functions modeling financial contexts, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship, or given input-output pairs including reading these from a table (e.g., utilize linear and exponential functions to compare simple with compound interest).
M.FAM.18	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. Data may address, but is not limited to automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.
M.FAM.19	Calculate and interpret the average rate of change of a function modeling a financial context (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph (e.g., examine depreciation trends).
M.FAM.20	Graph functions expressed symbolically and show key features of the graph (e.g., graph the linear, quadratic, or exponential curve that models the demand versus supply functions and find the equilibrium point with and without technology). Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.
M.FAM.21	Compare properties of two functions each represented in a different way, such as algebraically, graphically, numerically in tables, or by verbal descriptions (e.g., utilize linear and exponential functions to compare simple with compound interest; calculate and compare using both the loan payment formula and payment schedules in table format, the monthly cost of purchasing an automobile, and discuss the feasibility of that payment in relation to monthly budget; compare two functions showing interest accrued when paying the minimum monthly payment over time compared to paying a larger monthly payment; identify and compare the average rate of change between given time periods).
M.FAM.22	Graph linear and quadratic functions and show intercepts, maxima, and minima (e.g., in the model of a profit function, determine the break-even points, the maximum possible loss, and the maximum profit).
M.FAM.23	Write a function that describes a relationship between two quantities in a financial context (e.g., calculate the costs associated with purchasing a vehicle, including leasing, purchasing with cash, or with a loan).
M.FAM.24	Identify the effect on functions that model financial situations of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology (e.g., identify the impact of a change in a constraint in a function that models retirement planning, business income and expenses, or employment benefits).
M.FAM.25	Graph square root, cube root, and piecewise-defined functions that model financial situations, including step functions and absolute value functions (e.g., develop and communicate the appropriateness of representing a commission salary using a linear versus a piecewise function; analyze graphs of functions that model profit).
M.FAM.26	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model financial situations, and translate between the two forms (e.g., create recursive and explicit models of sequences related to retirement planning, amortization schedules for a loan, comparing subsidized and unsubsidized loans,

	reasoning and making decisions about the length of the loan and a fixed versus adjustable rate mortgage).
M.FAM.27	Apply exponential formulas to solve for future and present value of investments by hand or with graphing technology (e.g., $PV=FV*(1/(1+r)^n)$ and $A(t) = Pe^{rt}$).

Financial Modeling with Data

Cluster	**Represent, summarize, and evaluate data.**
M.FAM.28	Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots). Instructional Note: Data may address, but is not limited to automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.
M.FAM.29	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit regression lines to scatterplots and make predictions based on lines of best fit. Find and interpret correlation coefficients of regression equations in financial situations (e.g., use scatter plots to show correlation between two funds, two stocks, a stock and the general market, or in business situations to forecast sales or to compare revenue to the number of units sold).
M.FAM.30	Create a data display modeling financial situations. Instructional Note: This may include, but is not limited to, modeling the different savings options for a given investment at local banking establishments; calculating and comparing the monthly cost of purchasing an automobile using both the loan payment formula and payment schedules in table format; creating an amortization schedule through the use of spreadsheet technology and the formula tool for a loan given principle, term, monthly payment, and interest rate; creating representations of pay schedules using a variety of modeling technologies; and making decisions in a financial context based on those representations.
M.FAM.31	Summarize categorical data in various forms (e.g., two-way frequency tables, circle graphs, segmented bar charts). Interpret relative frequencies in the context of the data in making financial decisions.
M.FAM.32	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: Data sets may address, but are not limited to, automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.
M.FAM.33	Use units to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays (e.g., use units appropriately as a way to understand multi-step problems in relationship to understanding credit card fees and finance charges; compute distance, rate and time to solve problems to analyze driving and safety data, using single and multiple unit conversion; use and compare researched reaction times and vehicle velocity, as well as accepted equations to solve problems with braking distances).
M.FAM.34	Use financial models from automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning to solve problems.

M.FAM.35	Evaluate reports based on data. Data may address, but is not limited to, planning for retirement or stock markets.
M.FAM.36	Use probability and expected value to analyze financial situations (e.g., model and compare automobile insurance policies).
M.FAM.37	Evaluate the impact of taxes on business ownership including property tax, sales tax, social security, retirement, and disability benefits. Evaluate the impact of taxes on personal finance decisions.

Mathematics – Quantitative Reasoning

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Quantitative Reasoning prepares students to reason, model, and draw conclusions or make decisions with mathematical, statistical, and quantitative information. Students will compare, analyze, and synthesize of multiple forms or sources of quantitative information. Students will use appropriate mathematical and statistical language in oral, written, and graphical forms; read and interpret authentic texts such as advertisements, consumer information, government forms, and newspaper articles containing quantitative information, including graphical displays of quantitative information. Students will develop an answer to an open-ended question requiring analysis and synthesis of multiple calculations, data summaries, and/or models. Students will draw conclusions or make decisions in quantitatively based situations that are dependent upon multiple factors and analyze how different situations would affect the decisions. Students will be able to critique and evaluate quantitative arguments that utilize mathematical, statistical, and quantitative information. Students will evaluate the validity and possible biases in arguments presented in authentic contexts based on multiple sources of quantitative information (e.g., advertising, internet postings, consumer information, political arguments). The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Logical Reasoning	**Algebraic Modeling and Number Sense**
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	• Create and analyze mathematical models to make decisions related to earning, investing, spending, and borrowing money.
Descriptive Statistics	**Probability**
• Make decisions based on understanding, analysis, and critique of reported statistical information and summaries. • Use basic rules of counting and probability to analyze and evaluate risk and return in the context of everyday situations. • Interpret categorical and quantitative data, make inferences and justify conclusions. • Create and analyze mathematical models to make decisions related to earning, investing, spending, and borrowing money.	• Make conclusions based on understanding, analysis and critique of probabilities (e.g., compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer). • Develop a probability distribution (e.g., find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices; find the expected grade under various grading schemes).

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Quantitative Reasoning:

Logical Reasoning	
Mathematics as a language.	Standards 1-2
Tools for problem solving.	Standard 3
Algebraic Modeling	
Understand ratio concepts and use ratio reasoning to solve problems.	Standard 4
Work with integer exponents, scientific notation, and radicals.	Standards 5-7
Reason quantitatively and use units to solve problems.	Standard 8
Represent and solve equations and inequalities graphically.	Standard 9
Explain volume formulas and use them to solve problems.	Standard 10
Understand financial models.	Standards 11-13
Reason quantitatively and use units to solve problems.	Standards 14-15
Create equations that describe numbers or relationships.	Standards 16-17
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 18
Build a function that models a relationship between two quantities.	Standard 19
Interpret linear models.	Standard 20
Descriptive Statistics	
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standard 21
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 22-24
Perform statistical analysis.	Standard 25
Communicate statistical information.	Standards 26-27
Probability	
Analyze information using probability and counting.	Standards 28-29
Use probability to evaluate outcomes and manage uncertainty.	Standards 30-31
Understand independence and conditional probability and use them to interpret data.	Standards 32-36
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Standards 37-40
Use probability to evaluate outcomes of decisions.	Standards 41-42

Logical Reasoning

Cluster	Mathematics as a language.
M.QR.1	Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical arguments and analyze the soundness of mathematical arguments of others.
M.QR.2	Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.

Cluster	Tools for problem solving.
M.QR.3	Gather data, conduct investigations, and apply mathematical concepts and models to solve problems in mathematics and other disciplines.

Algebraic Modeling

Cluster	Understand ratio concepts and use ratio reasoning to solve problems.
M.QR.4	Use ratio and rate reasoning to solve real-world and mathematical problems. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed (e.g., if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?). c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Cluster	Work with integer exponents, scientific notation, and radicals.
M.QR.5	Know and apply the properties of integer exponents to generate equivalent numerical expressions.
M.QR.6	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
M.QR.7	Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Cluster	Reason quantitatively and use units to solve problems.
M.QR.8	Define appropriate quantities for the purpose of descriptive modeling.

Cluster	Represent and solve equations and inequalities graphically.
M.QR.9	Recognize that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

Cluster	Explain volume formulas and use them to solve problems.
M.QR.10	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Cluster	Understand financial models.
M.QR.11	Determine, represent and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., business loans, auto, mortgage, and/or credit card).
M.QR.12	Determine, represent and analyze mathematical models for investments involving simple and compound interest with and without additional deposits (e.g., savings accounts, bonds, and/or certificates of deposit).
M.QR.13	Research and analyze taxes including payroll, sales, personal property, real estate, and income tax returns.

Cluster	Reason quantitatively and use units to solve problems.
M.QR.14	Use units to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.QR.15	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	Create equations that describe numbers or relationships.
M.QR.16	Create equations and inequalities in one variable, representing linear, quadratic, simple rational, and exponential relationships, and use them to solve problems.
M.QR.17	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.QR.18	Construct linear and exponential functions, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship, or given input-output pairs (include reading these from a table).

Cluster	Build a function that models a relationship between two quantities.
M.QR.19	Write a function that describes a relationship between two quantities. a. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. b. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Cluster	Interpret linear models.
M.QR.20	Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit.

Descriptive Statistics

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.QR.21	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.QR.22	Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots).
M.QR.23	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets.
M.QR.24	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Cluster	Perform statistical analysis.
M.QR.25	Create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features.

Cluster	Communicate statistical information.
M.QR.26	Report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied.
M.QR.27	Communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language.

Probability

Cluster	Analyze information using probability and counting.
M.QR.28	Use the Fundamental Counting Principle, permutations and combinations to determine all possible outcomes for an event; determine probability and odds of a simple event; explain the significance of the Law of Large Numbers.
M.QR.29	Determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, two-way frequency tables, and area models, to make decisions in problem situations.

Cluster	Use probability to evaluate outcomes and manage uncertainty.
M.QR.30	Use probabilities to make and justify decisions about risks in everyday life.
M.QR.31	Calculate expected value to analyze mathematical fairness, payoff, and rise.

Cluster	Understand independence and conditional probability and use them to interpret data.
M.QR.32	Describe events as subsets of a sample space using characteristics of the outcomes or as unions, intersections, or complements of other events.
M.QR.33	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities. Use this characterization to determine if they are independent.
M.QR.34	Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
M.QR.35	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
M.QR.36	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Cluster	Use the rules of probability to compute probabilities of compound events in a uniform probability model.
M.QR.37	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.
M.QR.38	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model.
M.QR.39	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B) and interpret the answer in terms of the model.
M.QR.40	Use permutations and combinations to compute probabilities of compound events and solve problems.

Cluster	Use probability to evaluate outcomes of decisions.
M.QR.41	Use probabilities to make fair decisions.
M.QR.42	Analyze decisions and strategies using probability concepts.

Mathematics – Statistics

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Knowledge of topics related to probability and statistics is critical to decision-making and to the analysis of data. The Statistics course is designed to be a full-year course that addresses all standards and provides an opportunity to address the fundamental ideas and most commonly used techniques to organize and make sense of data. This course builds on knowledge of probability, randomness, and variability to provide students with an understanding of experimental design, estimation, hypothesis testing, and effective communication of experimental results. Using concepts of probability and statistics, students predict the likelihood of an event occurring, organize and evaluate data, and identify the significance of statements. Students investigate types of probability, determine probability and odds using multiple counting principles and distributions, and apply the concepts to real-world problems. Statistical information collected and analyzed by students is used to investigate ways of collecting, displaying, and analyzing data. Students analyze and justify using statistical concepts to test validity of a hypothesis and of correlation as applied in real-world situations. Students identify a real-world situation that involves statistical concepts, make a hypothesis as to the outcome; develop, justify, and implement a method to collect, organize, and analyze data; generalize the results to make a conclusion, compare the hypothesis and the conclusion; present their findings using predictive and analytic tools in a clear and concise manner. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Descriptive Statistics	**Probability**
• Given a two-way table of relative frequencies that summarizes survey data, relate a person's highest level of education and their role model to determine if a person whose highest level of education is a bachelor's degree is more likely to have a family member than a stranger as a role model.	• A cereal company is putting a prize in each box of cereal. The company is offering four different and evenly distributed prizes. How many boxes should one expect to need to buy to get all four prizes?
Probability Distributions	**Correlation and Regression**
• The heights of five women are measured to be 63 inches, 68 inches, 56 inches, 64 inches, and 67 inches. Determine the expected value of the height of a randomly chosen woman.	• A random sample of beef hotdogs was taken and the amount of sodium (in mg) and calories were measured. Use the provided information and the regression equation to determine the amount of sodium in a beef hotdog with 170 calories and in a beef hotdog with 120 calories. Which of the calculated sodium levels is closer to the true sodium level? Why?

Confidence Intervals	Hypothesis Testing with One Variable
• Alyssa has over 500 songs saved on her phone. She wants to estimate the proportion of songs by a female artist. After taking a simple random sample of 50 songs, she finds that 20 of the sampled songs are by a female artist. Determine a 99% confidence interval for the proportion of songs on her phone that are by a female artist.	• Ellen has a pair of dogs and she noticed that they seem to breed more male puppies than female puppies. In the next litter of 12 puppies, there were 9 male puppies. Test the hypothesis that each puppy has an equal chance of 50% of being either male or female versus the alternative that the chance of a male puppy is greater. Look at the results of 1000 simulations, each simulating 12 puppies with a 50% chance of being male or female. According to the simulations, what is the probability of having 9 male puppies or more out of 12? If the observed outcome has a probability less than 1% under the tested hypotheses, reject the hypothesis. What should be concluded regarding the hypothesis?
Statistical Inference	
• A researcher wants to know if the children from three schools have equal mean IQ scores. Each school enrolls 1000 students. But there is neither the time nor funding to test all 3000 students. Based on a simple random survey of 10 students from each school, perform and analyze an ANOVA test.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Statistics:

Descriptive Statistics	
Summarize, represent, and interpret data on single count or measurement variable.	Standards 1-8
Probability	
Understand independence and conditional probability and use them to interpret data.	Standards 9-17
Probability Distributions	
Calculate expected values and use them to solve problems.	Standards 18-24
Correlation and Regression	
Interpret linear models.	Standards 25-27
Confidence Intervals	
Determine and interpret confidence intervals.	Standards 28-33
Hypothesis Testing with One Variable	
Use hypothesis testing in making and interpreting decisions.	Standards 34-42
Statistical Inference	
Determine and use correlation.	Standards 43-46
Use linear regression to predict and interpret.	Standards 47-51
Use statistical tests to determine a relationship.	Standards 52-57

Descriptive Statistics

Cluster	Summarize, represent, and interpret data on single count or measurement variable.
M.PS.1	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
M.PS.2	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
M.PS.3	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
M.PS.4	Evaluate reports based on data. Write a function that describes a relationship between two quantities.
M.PS.5	Represent data with plots on the real number line (dots plots, histograms, and box plots).
M.PS.6	Use statistics appropriate to the shape of the data distributions to compare center and spread of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
M.PS.7	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
M.PS.8	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Probability

Cluster	Understand independence and conditional probability and use them to interpret data.
M.PS.9	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
M.PS.10	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.
M.PS.11	Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
M.PS.12	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

M.PS.13	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
M.PS.14	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.
M.PS.15	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model.
M.PS.16	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B) and interpret the answer in terms of the model.
M.PS.17	Use permutations and combinations to compute probabilities of compound events and solve problems.

Probability Distributions

Cluster	**Calculate expected values and use them to solve problems.**
M.PS.18	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
M.PS.19	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
M.PS.20	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated to find the expected value.
M.PS.21	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically to find the expected value.
M.PS.22	Weight the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values (e.g., find the expected payoff for a game of chance).
M.PS.23	Evaluate and compare strategies on the basis of expected values.
M.PS.24	Analyze decisions and strategies using probability concepts.

Correlation and Regression

Cluster	**Interpret linear models.**
M.PS.25	Represent data on two quantitative variables on a scatter plot and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. Focus should be on situations for which linear models are appropriate. c. Fit a linear function for scatter plots that suggest a linear association.
M.PS.26	Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit. Instructional Note: The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.PS.27	Distinguish between correlation and causation.

Confidence Intervals

Cluster	Determine and interpret confidence intervals.
M.PS.28	Find the point estimate and margin of error in a given scenario.
M.PS.29	Construct and interpret confidence intervals for the population mean.
M.PS.30	Determine minimum sample size requirements when estimating mean, μ (population proportion).
M.PS.31	Interpret the t-distribution and use t-distribution table in real-world scenarios.
M.PS.32	Construct confidence intervals when the sample size, n, is less than 30, population is normally distributed, and standard deviation, σ, is unknown.
M.PS.33	Interpret the chi-square distribution and use chi-square distribution table. Use the chi-square distribution to construct a confidence interval for the variance and standard deviation.

Hypothesis Testing with One Variable

Cluster	Use hypothesis testing in making and interpreting decisions.
M.PS.34	Interpret a hypothesis test; state a null hypothesis and an alternative hypothesis.
M.PS.35	Identify Type I and Type II errors and interpret the level of significance.
M.PS.36	Use one-tailed and two-tailed statistical tests to find p-value.
M.PS.37	Make and interpret decisions on comparing two hypotheses based on results of a statistical test. Write a claim for a hypothesis test.
M.PS.38	Find probability values and test for mean. Use in a z-test.
M.PS.39	Find critical values and rejection regions in a normal distribution. Use rejection regions for a z-test.
M.PS.40	Find critical values in a t-distribution and use the t-test to test a mean.
M.PS.41	Use the z-test to test a population proportion, p.
M.PS.42	Find critical values for chi squared test. Use the chi squared test to test a variance or a standard deviation.

Statistical Inference

Cluster	Determine and use correlation.
M.PS.43	Find a correlation coefficient.
M.PS.44	Test a population correlation coefficient using a table.
M.PS.45	Perform a hypothesis test for a population correlation coefficient.
M.PS.46	Distinguish between correlation and causation.

Cluster	Use linear regression to predict and interpret.
M.PS.47	Find the equation of a regression line; predict y-values using a regression line.
M.PS.48	Interpret the types of variation about a regression line.
M.PS.49	Find and interpret the coefficient of determination.
M.PS.50	Find and interpret the standard error of estimate for a regression line; construct and interpret a prediction interval for y.
M.PS.51	Use technology to find a multiple regression equation, the standard error of estimate, and the coefficient of determination.

Cluster	Use statistical tests to determine a relationship.
M.PS.52	Use a contingency table to find expected frequencies.
M.PS.53	Use the chi-squared distribution to test whether a frequency distribution fits a claimed distribution and to test whether two variables are independent.
M.PS.54	Interpret the F-distribution and use an F-table to find critical values.
M.PS.55	Perform a two-sample F-test to compare two variances.
M.PS.56	Perform a two-sample F-test to compare two variances. Interpret the F-distribution and use an F-table to find critical values.
M.PS.57	Use one-way analysis of variance to test claims involving three or more means. Introduce two-way analysis of variance.

Mathematics – Introduction to Mathematical Applications

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Introduction to Mathematical Applications will solidify their quantitative literacy by enhancing numeracy and problem-solving skills as they investigate and use fundamental concepts of algebra, geometry, and statistical analysis to apply to authentic career projects and scenarios. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Number and Quantity	**Algebra: Seeing Structure in Expressions**
• Develop an understanding of basic operations, equivalent representations, and properties of the real number system.	• Create equations or inequalities that model physical situations.
Functions: Interpreting Functions	**Geometry/Trigonometry**
• Develop knowledge and understanding of the concept of functions as they use, analyze, represent, and interpret functions and their applications.	• Solve application problems by calculating area or surface area in a two-dimensional object or volume in three-dimensional objects. • Understand and apply the Pythagorean Theorem for solving real-world problems (e.g., checking accuracy on gate construction, conduit bending).
Modeling	**Statistics: Interpreting Categorical & Quantitative Data**
• Create and use two- and three-dimensional representations of authentic situations in problem solving. • Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	• Analyze and interpret tables, charts, and graphs (e.g., interpret a body mass index (BMI) chart). • Distinguish between correlation and causation.
Finance Mathematics	
• Determine, represent, and analyze mathematical models for personal finance.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Introduction to Mathematical Applications:

Number and Quantity	
Mathematics as a language.	Standards 1-3
Mathematics and Measurement.	Standards 4-7

The Real Number System.	Standards 8-9
Algebra: Seeing Structure in Expressions	
Understand the connections between proportional relationships, lines, and linear equations.	Standards 10-11
Create equations that describe numbers or relationships.	Standards 12-15
Solve systems of equations.	Standard 16
Functions: Interpreting Functions	
Understand the concept of a function and use function notation.	Standard 17
Analyze functions using different representations.	Standards 18-20
Build a function that models a relationship between two quantities.	Standards 21-22
Geometry/Trigonometry	
Visualize relationships between two-dimensional and three-dimensional objects and apply geometric concepts in modeling situations.	Standards 23-24
Use geometric theorems and formulas to solve problems.	Standards 25-29
Modeling	
Concrete geometric representation (physical modeling).	Standards 30-31
Summarize, represent, and interpret data on two quantitative variables.	Standards 32-34
Statistics and Probability: Interpreting Categorical & Quantitative Data	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 35-39
Finance Mathematics	
Understand financial models.	Standards 40-41
Personal use of finance.	Standards 42-43

Number and Quantity

Cluster	**Mathematics as a language.**
M.IMA.1	Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical arguments and analyzing the soundness of mathematical arguments of others.
M.IMA.2	Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.
M.IMA.3	Use units to understand problems and to guide the solutions of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Cluster	Mathematics and Measurement.
M.IMA.4	Select and correctly use an appropriate tool (e.g., tape measure, ruler, compass, level, micrometer, scale, protractor, thermometer, speedometer, odometer, pressure gauge, measuring squares, multimeter) to measure and/or calculate lengths, distances, directions, masses, temperatures, rates of change (e.g., slope, speed), areas, volumes, voltages, currents, and resistances.
M.IMA.5	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
M.IMA.6	Solve real-world problems requiring conversion of units using dimensional analysis for measurements in English and metric systems. Solve problems involving multiple units of measurement (e.g., converting between currencies, calculating dosages of medicine, trip planning from miles to kilometers).
M.IMA.7	Distinguish between proportional and non-proportional situations, apply proportional reasoning when appropriate, solve for an unknown quantity in proportional situations; apply scale factors to perform indirect measurements using maps, blueprints, concentrations, dosages, and densities.

Cluster	The Real Number System.
M.IMA.8	Perform operations and convert quantities between fractions, decimals, and percents using positive and negative numbers, fractions, absolute value, decimals, percentages, and scientific notation (e.g., given the cost of a project, determine what percentage of the budget were salaries; percent of increase/decrease).
M.IMA.9	Solve real-world problems in a variety of contexts by representing quantities in equivalent forms (fractions, decimals, and percentages) to investigate and describe quantitative relationships. Compare the size of numbers in different forms arising in authentic real-world contexts, such as growth expressed as a fraction versus as a percentage. Interpret the meaning of numbers in different forms, such as scientific notation and the meaning of a fraction or percentage greater than 100 and its validity in a given context. Recognize incorrect or deceptive uses of fractions, decimals, or percentages.

Algebra – Seeing Structure in Expressions

Cluster	Understand the connections between proportional relationships, lines, and linear equations.
M.IMA.10	Graph proportional relationships, interpreting the unit rates as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed (e.g., labor cost per time, material cost per job).
M.IMA.11	Solve application problems using direct and inverse variation equations (e.g., determine the mechanical advantage of gears, Ohm's Law).

Cluster	Create equations that describe numbers or relationships.
M.IMA.12	Analyze real-world problem situations and use variables to construct and solve equations involving one or more unknown or variable quantities to answer questions about the situations, such as creating spreadsheet formulas to calculate prices based

	on percentage mark-up or solving formulas for specified values. Demonstrate understanding of the meaning of a solution. Identify when there is insufficient information given to solve a problem.
M.IMA.13	Analyze real-world problem situations and use variables to construct and solve equations and inequalities in one variable, representing linear, exponential, and simple rational functions (e.g., using spreadsheet functions, determine sale price of items).
M.IMA.14	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales (e.g., profit vs. number of units, cost vs. number of units, resistance vs. current).
M.IMA.15	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations (e.g., rearrange Ohm's law V = IR to highlight resistance R).

Cluster	**Solve systems of equations.**
M.IMA.16	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables (e.g., childcare facility – sq. footage to number of children; solving electrical current in a circuit with multiple paths, break-even point).

Functions – Interpreting Functions

Cluster	**Understand the concept of a function and use function notation.**
M.IMA.17	Use multiple representations of functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically.

Cluster	**Analyze functions using different representations.**
M.IMA.18	Interpret the parameters in a linear function in terms of a context.
M.IMA.19	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
M.IMA.20	Describe qualitatively the functional relationship between two quantities by analyzing a graph.

Cluster	**Build a function that models a relationship between two quantities.**
M.IMA.21	Represent application problems as linear equations. Write a function that describes a relationship between two quantities (e.g., level of education versus pay; rate of speed versus fuel consumption; caloric intake versus expenditure).
M.IMA.22	Recognize that the graph of a linear or exponential equation in two variables is the set of all its solutions plotted in the coordinate plane.

Geometry/Trigonometry

Cluster	Visualize relationships between two dimensional and three-dimensional objects and apply geometric concepts in modeling situations.
M.IMA.23	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects (e.g., three-view drawings and blueprints).
M.IMA.24	Use two- and three-dimensional shapes and circles, their measures, and their properties to describe objects. a. Apply concepts of density based on area and volume in modeling situations. b. Apply geometric methods to solve design problems to satisfy given constraints (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios).

Cluster	Use geometric theorems and formulas to solve problems.
M.IMA.25	Explore theorems about triangles to solve real-world application problems.
M.IMA.26	Understand and apply the Pythagorean Theorem for solving real-world problems (e.g., checking accuracy on gate construction, conduit bending, roof pitch). Instructional Note: If students have had experience and exposure to right triangle trigonometry, extend this standard to apply trigonometric ratios and special right triangle relationships, 30-60-90 and 45-45-90, to solve right triangles in applied problems.
M.IMA.27	Solve application problems by calculating area and surface area for two-dimensional objects (e.g., calculate the cost of installing flooring in a building and painting the interior and exterior of a building based on square footage).
M.IMA.28	Solve application problems by calculating volume for three-dimensional objects using formulas for cylinders, pyramids, prisms, cones, and spheres (e.g., compute amount of cement needed for a sidewalk, amount of water in a fire hose, amount of air in ductwork).
M.IMA.29	Solve application problems by calculating circumference, area, radius, diameter, area of sector, arc length of a circle with appropriate unit labels (e.g., develop a circular watering system).

Modeling

Cluster	Concrete geometric representation (physical modeling)
M.IMA.30	Create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications.
M.IMA.31	Gather data, conduct investigations, and apply mathematical concepts and models to solve problems (e.g., designing and building a house or a car).

Cluster	Summarize, represent, and interpret data on two quantitative variables.
M.IMA.32	Collect numerical bivariate data; represent data on two quantitative variables on a scatter plot; determine whether or not a relationship exists; if so, describe how the variables are related and select a function to model the data, justify the selection and use the model to make predictions (e.g., cost of the materials for a construction

	project, cost of the labor for a project, cost and value of a vehicle based on depreciation).
M.IMA.33	For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. Interpret the rate of change and the constant term of a linear model in the context of the data.
M.IMA.34	Identify positive and negative correlations (e.g., vehicle depreciation). Use technology to compute and interpret the correlation coefficient of a linear fit. Instructional Note: The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.

Statistics and Probability: Interpreting Categorical & Quantitative Data

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**
M.IMA.35	Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots).
M.IMA.36	Analyze and interpret tables, charts, and graphs (e.g., interpret a body mass index (BMI) chart).
M.IMA.37	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets.
M.IMA.38	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
M.IMA.39	Distinguish between correlation and causation.

Finance Mathematics

Cluster	**Understanding financial models.**
M.IMA.40	Determine, represent, and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., business loans, auto, mortgage, and/or credit card).
M.IMA.41	Determine, represent, and analyze mathematical models for investments involving simple and compound interest with and without additional deposits (e.g., savings accounts, bonds, and/or certificates of deposit).

Cluster	**Personal use of finance.**
M.IMA.42	Research, develop, and analyze personal budgets based on given parameters (e.g., fixed and discretionary expenses, insurance, gross vs. net pay, types of income, wage, salary, commission, career choice, geographic region, retirement, and/or investment planning).
M.IMA.43	Research and analyze taxes including payroll, sales, personal property, real estate, and income tax returns.

Appendix A: High School Mathematics I and Mathematics II Standards

High School Mathematics I Standards				
Algebra I Standards			Geometry Standards	
M.A1HS.1	M.A1HS.16	M.A1HS.27	M.GHS.1	M.GHS.8
M.A1HS.2	M.A1HS.17	M.A1HS.28	M.GHS.2	M.GHS.9
M.A1HS.3	M.A1HS.18	M.A1HS.29	M.GHS.3	M.GHS.10
M.A1HS.4	M.A1HS.19	M.A1HS.30	M.GHS.4	M.GHS.11
M.A1HS.5	M.A1HS.20	M.A1HS.31	M.GHS.5	M.GHS.12
M.A1HS.7	M.A1HS.21	M.A1HS.32	M.GHS.6	M.GHS.13
M.A1HS.8	M.A1HS.22	M.A1HS.33	M.GHS.7	M.GHS.16
M.A1HS.9	M.A1HS.23	M.A1HS.34		
M.A1HS.10	M.A1HS.24	M.A1HS.35		
M.A1HS.12	M.A1HS.25	M.A1HS.36		
M.A1HS.13	M.A1HS.26	M.A1HS.37		
M.A1HS.14				

High School Mathematics II Standards				
Algebra I Standards		Geometry Standards		
M.A1HS.1	M.A1HS.23	M.GHS.14	M.GHS.26	M.GHS.38
M.A1HS.2	M.A1HS.24	M.GHS.15	M.GHS.27	M.GHS.39
M.A1HS.5	M.A1HS.25	M.GHS.16	M.GHS.28	M.GHS.40
M.A1HS.6	M.A1HS.26	M.GHS.17	M.GHS.29	M.GHS.41
M.A1HS.11	M.A1HS.28	M.GHS.18	M.GHS.30	M.GHS.42
M.A1HS.15	M.A1HS.29	M.GHS.19	M.GHS.31	M.GHS.43
M.A1HS.22		M.GHS.20	M.GHS.32	M.GHS.44
		M.GHS.21	M.GHS.33	M.GHS.45
		M.GHS.22	M.GHS.34	M.GHS.46
		M.GHS.23	M.GHS.35	M.GHS.47
		M.GHS.24	M.GHS.36	M.GHS.48
		M.GHS.25	M.GHS.37	

Appendix B: High School Algebra I for 8th Grade or High School Mathematics I for 8th Grade

To ensure students are provided the opportunity to master the standards found in Mathematics 8, when offering high school mathematics credit during the middle school experience, county boards of education must submit a plan to the West Virginia Department of Education's Office of Teaching and Learning indicating the integration of grade 7, grade 8, and High School Algebra I/Mathematics I standards into their county-wide middle school mathematics programs. This plan will allow middle schools to offer credit-bearing high school courses in either High School Algebra I or High School Mathematics I to 8th grade students while ensuring students are taught grade 8 standards during middle school.