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MISSISSIPPI DEPARTMENT OF EDUCATION

Ensuring a bright future for every child

2019 Mississippi Alternate Academic Achievement Standards for Mathematics

Effective Date: 2019-2020 School Year

2019 Mississippi Alternate Academic Achievement Standards for Mathematics

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Introduction

The Mississippi Department of Education (MDE) is dedicated to student success, which includes improving student achievement in mathematics and establishing communication skills within a technological environment. The *2019 Mississippi Alternate Academic Achievement Standards* (MS AAAS) provide a consistent, clear understanding of what students are expected to know and be able to do by the end of each grade level or course. The purpose of the alternate standards is to build a bridge from the content in the general education mathematics framework to academic expectations for students with the most significant cognitive disabilities. The standards are designed to be rigorous and relevant to the real world, reflecting the knowledge and skills that students need for success in postsecondary settings.

In special education, prompting is often used to mean a system of structured cues to elicit desired behaviors that otherwise would not occur. In order to clearly communicate that teacher assistance is permitted during instruction of the MS AAAS and is not limited to structured prompting procedures. Guidance and support during instruction should be interpreted as teacher encouragement, general assistance, and informative feedback to support the student.

Purpose

In an effort to closely align instruction for students with significant cognitive disabilities who are progressing toward individualized postsecondary goals, the *MS AAAS for Mathematics* includes grade- and course-specific standards for grades K-12 mathematics. These standards are intended solely for students who have met the criteria for a Significant Cognitive Disability (SCD) as documented in each student's individualized education program (IEP).

This document is designed to provide special education teachers with a basis for curriculum development. As such, this set of alternate standards addresses a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. This framework outlines what knowledge students should obtain and the types of skills students should demonstrate upon completion. The *MS AAAS* are aligned to the Mississippi College- and Career-Readiness Standards (MS CCRS).

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking**; **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

(K-8) Mathematics Domain and (HS) Conceptual Category

Core Idea

Mississippi College- and Career-Readiness Standards (MS CCRS)

Mississippi Alternate Academic Achievement Standards (MS AAAS)

Grade K

Counting and Cardinality (CC)	
Know number names and the count sequence	
K.CC.1. Count to 100 by ones and by tens.	**A.K.CC.1.** Using vocalization, sign language, augmentive communication, or assistive technology, count to 10 by ones starting with one.
K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	Not applicable. Addressed in **A.2.NBT.2.b.**
K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	Not applicable. Addressed in **A.2.NBT.3.**

Implementation

The *2019 MS AAAS for Mathematics* will be implemented beginning in the 2019-2020 school year.

Technology

The MDE strongly encourages the use of technology in all mathematics classrooms. Technology is essential in teaching and learning mathematics; it influences the mathematics taught and enhances student learning. Calculators are often an allowable accommodation. Please consider students' individual learning needs when using technology in the classroom.

The following resources served as a foundation for the development of the *2019 MS AAAS for Mathematics*:

- *Mississippi's College and Career Readiness Standards (MS CCRS) for Mathematics*.
- Dynamic Learning Maps Consortium. (2013). *Dynamic Learning Maps Essential Elements for Mathematics*. Lawrence, KS: University of Kansas.

The Mississippi Alternate Academic Achievement Standards are based on the Dynamic Learning Maps Essential Elements (DLM EE), with additional edits and clarifications to better support the needs of Mississippi teachers and students. Standards language in italicized font indicates Mississippi-specific standards or adjustments to the DLM EE.

Alternate Academic Achievement Standards for Mathematics (Grades K-5)

Fluency/Fluently Defined

Throughout the *2019 MS AAAS for Mathematics* Grades K-5 standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: ***fluently***). With respect to student performance and effective in-class instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend one or more grades earlier in the standards than the grade when fluency is finally expected.

Wherever the word *fluently* appears in an *MS AAAS* content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow—fluent isn't halting, stumbling, or reversing oneself.

A key aspect of fluency in this sense is that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade level to allow all students to meet the standards that call explicitly for fluency.

Grade K

In kindergarten, instruction should focus on two critical areas: (1) representing, relating, and operating on whole numbers—initially with sets of objects; and (2) describing shapes and space. More learning time in kindergarten should be devoted to numbers than to other topics. Each critical area is described below.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5 (kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged but is not required). Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three- dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

(3) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade K

<table>
<tr><th colspan="2">Counting and Cardinality (CC)</th></tr>
<tr><td colspan="2">Know number names and the count sequence</td></tr>
<tr><td>K.CC.1. Count to 100 by ones and by tens.</td><td>A.K.CC.1. Using vocalization, sign language, augmentive communication, or assistive technology, count to 10 by ones starting with one.</td></tr>
<tr><td>K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).</td><td>Not applicable. Addressed in A.2.NBT.2.b.</td></tr>
<tr><td>K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).</td><td>Not applicable. Addressed in A.2.NBT.3.</td></tr>
<tr><td colspan="2">Count to tell the number of objects</td></tr>
<tr><td>K.CC.4. Understand the relationship between numbers and quantities; connect counting to cardinality.</td><td rowspan="4">A.K.CC.4. Demonstrate one-to-one correspondence, pairing each object with one, and only one, number and each number with one, and only one, object.</td></tr>
<tr><td>K.CC.4.a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.</td></tr>
<tr><td>K.CC.4.b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.</td></tr>
<tr><td>K.CC.4.c. Understand that each successive number name refers to a quantity that is one larger.</td></tr>
<tr><td>K.CC.5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.</td><td>A.K.CC.5. Using vocalization, sign language, augmentive communication, or assistive technology, count out up to three objects from a larger set, pairing each object with one, and only one, number name to tell how many.</td></tr>
<tr><td colspan="2">Compare numbers</td></tr>
<tr><td>K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]</td><td>A.K.CC.6. Identify whether the number of objects in one group is more or less than (e.g., when the quantities are clearly different) or equal to the number of objects in another group.</td></tr>
<tr><td>K.CC.7. Compare two numbers between 1 and 10 presented as written numerals.</td><td>Not applicable. Addressed in A.2.NBT.4.</td></tr>
</table>

[1] Include groups with up to 10 objects.

Operations and Algebraic Thinking (OA)

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from

K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings[3], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	**A.K.OA.1.** *Demonstrate an understanding of* addition as "putting together" or subtraction as "taking from" in everyday activities.
K.OA.2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	Not applicable. Addressed in **A.2.NBT.6–7**.
K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).	Not applicable. Addressed in **A.1.NBT.6**.
K.OA.4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	Not applicable. Addressed in **A.1.NBT.2**.
K.OA.5. Fluently add and subtract within 5.	Not applicable. Addressed in **A.3.OA.4**.

Number and Operations in Base Ten (NBT)

Work with numbers 11-19 to gain foundations for place value

K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	Not applicable. Addressed in **A.1.NBT.4.** and **A.1.NBT.6**.

Measurement and Data (MD)

Describe and compare measurable attributes

<table>
<tr><td>K.MD.1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.</td><td rowspan="2">A.K.MD.1-3. Classify objects according to attributes (e.g., big/small, heavy/light, tall/short).</td></tr>
<tr><td>K.MD.2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children, and describe one child as taller/shorter.</td></tr>
</table>

Classify objects and count the number of objects in each category	
K.MD.3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[2]	**A.K.MD.1-3.** Classify objects according to attributes (*e.g.,* big/small, heavy/light, *tall/short*).
Geometry (G)	
Identify and describe shapes (e.g., squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, spheres)	
K.G.1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.	Not applicable. Addressed in **A.1.G.a**.
K.G.2. Correctly name shapes regardless of their orientations or overall size.	**A.K.G.2–3.** Match shapes of the same size and orientation (*e.g.,* circle, square, rectangle, triangle).
K.G.3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	
Analyze, compare, create, and compose shapes	
K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	Not applicable. Addressed in **A.7.G.1**.
K.G.5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	Not applicable.
K.G.6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	Not applicable. Addressed in **A.1.G.3**.

[2] Limit category counts to less than or equal to 10.

Grade 1

In Grade 1, instruction should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes. Each critical area is described below.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use various models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting to two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop an understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[3]

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build an understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

(5) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards

[3] Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 1

Operations and Algebraic Thinking (OA)	
Represent and solve problems involving addition and subtraction	
1.OA.1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**A.1.OA.1.a**. Represent addition and subtraction with*in five using* objects, fingers, mental images, drawings, sounds (e.g., claps), or acting out situations.
	A.1.OA.1.b. Recognize two groups that have the same or equal quantity.
1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**A.1.OA.2.** *Demonstrate* "putting together" *two sets of objects* to solve *the* problem.
Understand and apply properties of operations and the relationship between addition and subtraction	
1.OA.3. Apply properties of operations as strategies to add and subtract.[4] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*	Not applicable. Addressed in **A.6.A.3.** and **A.N-CN.2**.
1.OA.4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*	Not applicable. Addressed in **A.1.NBT.4.** and **A.1.NBT.6**.
Add and subtract within 20	
1.OA.5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	**A.1.OA.5.a.** Use manipulatives or visual representations to indicate the number that results when adding one more.
	A.1.OA.5.b. Apply knowledge of "one less" to subtract one from a number.

[4] Students need not use formal terms for these properties.

1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	Not applicable. Addressed in **A.3.OA.4**.
Work with addition and subtraction equations	
1.OA.7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*	Not applicable. Addressed in **A.1.OA.1.b.** and **A.2.NBT.5.a**.
1.OA.8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 =* D *– 3, 6 + 6 =* D.	Not applicable. Addressed in **A.3.OA.4**.

Number and Operations in Base Ten (NBT)

Extend the counting sequence	
1.NBT.1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	**A.1.NBT.1.a.** Count by ones to 30.
	A.1.NBT.1.b. Count as many as 10 objects and represent the quantity with the corresponding numeral.
Understand place value	
1.NBT.2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:	**A.1.NBT.2.** Create sets of 10.
1.NBT.2.a. 10 can be thought of as a bundle of ten ones—called a "ten."	
1.NBT.2.b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.	
1.NBT.2.c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	

1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**A.1.NBT.3.** *Using vocalization, sign language, augmentive communication, or assistive technology,* compare two groups of 10 or fewer items *using appropriate vocabulary (e.g., more, less, equal)* when the number of items in each group is similar.
Use place value understanding and properties of operations to add and subtract	
1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	**A.1.NBT.4.** Compose numbers less than or equal to five in more than one way.
1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	Not applicable. Addressed in **A.1.OA.5.a.** and **A.1.OA.5.b**.
1.NBT.6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**A.1.NBT.6.** Decompose numbers less than or equal to five in more than one way.
Measurement and Data (MD)	
Measure lengths indirectly and by iterating length units	
1.MD.1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.	**1.MD.1–2**. Compare lengths to identify which is longer/shorter, taller/shorter.
1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same- size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	

Tell and write time with respect to a clock and a calendar	
1.MD.3. Tell and write time in hours and half-hours using analog and digital clocks. **1.MD.3.a**. Tell and write time in hours and half-hours using analog and digital clocks.	**A.1.MD.3.a.** Demonstrate an understanding of the terms tomorrow, yesterday, and today.
1.MD.3.b. Identify the days of the week, the number of days in a week, and the number of weeks in each month.	**A.1.MD.3.b.** Demonstrate an understanding of the terms morning, afternoon, day, and night.
	A.1.MD.3.c. Identify activities that come before, next, and after.
	A.1.MD.3.d. Demonstrate an understanding that telling time is the same every day.
Represent and interpret data	
1.MD.4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**A.1.MD.4.** Organize data into categories by sorting.
Work with money	
1.MD.5.a. Identify the value of all U.S. coins (penny, nickel, dime, quarter, half-dollar, and dollar coins). Use appropriate cent and dollar notation (e.g., 25¢, $1).	***A1.MD.5.a-d**. Using vocalization, sign language, augmentive communication, or assistive technology, identify U.S. coins by name (e.g., penny, nickel, dime, quarter).*
1.MD.5.b. Know the comparative values of all U.S. coins (e.g., a dime is of greater value than a nickel).	
1.MD.5.c. Count like U.S. coins up to the equivalent of a dollar.	
1.MD.5.d. Find the equivalent value for all greater value U.S. coins using like value smaller coins (e.g., 5 pennies equal 1 nickel; 10 pennies equal dime, but not 1 nickel and 5 pennies equal 1 dime).	

Geometry (G)	
Reason with shapes and their attributes	
1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	**A.1.G.1.** Identify the basic attributes of objects *(e.g., color, overall size)*.
1.G.2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[5]	**A.1.G.2.** Sort shapes of the same size and orientation (*e.g.,* circle, square, rectangle, triangle).
1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as *two of* or *four of* the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	**A.1.G.3.** Put *two pieces together* to make a shape that relates to the whole (*e.g.,* two semicircles to make a circle, two squares to make a rectangle).

[5] Students do not need to learn formal names such as "right rectangular prism."

Grade 2

In Grade 2, instruction should focus on four critical areas: (1) extending understanding of base-10 notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. Each critical area is described below.

(1) Students extend their understanding of the base-10 system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1,000) written in base-10 notation, recognizing that the digits in each place represent the amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1,000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-10 notation using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measurement (e.g., centimeter, inch), and they use rulers and other measurement tools with the understanding that linear measurement involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

(5) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 2

Operations and Algebraic Thinking (OA)	
Represent and solve problems involving addition and subtraction	
2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Not applicable. Addressed in **A.3.OA.4**.
Add and subtract within 20	
2.OA.2. Fluently add and subtract within 20 using mental strategies[6]. By end of Grade 2, know from memory all sums of two one-digit numbers.	Not applicable. Addressed in **A.2.NBT.6–7.** and **A.3.OA.4**.
Work with equal groups of objects to gain foundations for multiplication	
2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	**A.2.OA.3.** Equally distribute even numbers of objects between two groups.
2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	**A.2.OA.4.** Use *repeated* addition to find the *sum* of objects arranged *in* equal groups up to 10.
Number and Operations in Base Ten (NBT)	
Understand place value	
2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: 100 can be thought of as a bundle of ten tens — called a "hundred." The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	**A.2.NBT.1.** Represent numbers up to 30 with sets of tens and ones, using objects in columns or arrays.

[6] See standard 1.OA.C.6. for a list of mental strategies.

2.NBT.2. Count within 1000; skip-count by 5s starting at any number ending in 5 or 0. Skip-count by 10s and 100s starting at any number.	**A.2.NBT.2.a.** *Using vocalization, sign language, augmentive communication, or assistive technology,* count from 1 to 30 (count with meaning; cardinality).
	A.2.NBT.2.b. *Using vocalization, sign language, augmentive communication, or assistive technology,* name the next number in a sequence between 1 and 10.
2.NBT.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	**A.2.NBT.3.** Identify numerals 1 to 30.
2.NBT.4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	**A.2.NBT.4.** *Using vocalization, sign language, augmentive communication, or assistive technology,* compare sets of objects and numbers using appropriate vocabulary (*e.g.,* more, less, equal).
Use place value understanding and properties of operations to add and subtract	
2.NBT.5. ***Fluently*** add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	**A.2.NBT.5.a.** Identify the meaning of the "+" sign (i.e., combine, plus, add), "–" sign (i.e., separate, subtract, take), and the "=" sign (equal).
	A.2.NBT.5.b. Using concrete examples, compose and decompose numbers up to 10 in more than one way.
2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations.	**A.2.NBT.6-7.** Use objects, representations, and numbers (0–20) to add and subtract.
2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	
2.NBT.8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.	Not applicable.
2.NBT.9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[7]	Not applicable.

[7] Explanations may be supported by drawings or objects.

Measurement and Data (MD)

Measure and estimate lengths in standard units	
2.MD.1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	**A.2.MD.1.** Measure the length of objects using non-standard units.
2.MD.2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	Not applicable.
2.MD.3. Estimate lengths using units of inches, feet, centimeters, and meters.	**A.2.MD.3–4.** Order by length using non-standard units.
2.MD.4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	
Relate addition and subtraction to length	
2.MD.5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	**A.2.MD.5.** Increase or decrease length by adding or subtracting unit(s).
2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.	**A.2.MD.6.** Use a number line to add one more *or one less* unit of length.
Work with time with respect to a clock and a calendar, and work with money	
2.MD.7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	**A.2.MD.7**. Identify on a digital clock the hour that matches a routine activity.
2.MD.8a. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*	**A.2.MD.8.** *Identify the* value *of money (e.g., a penny has a value of 1 cent, a nickel has a value of 5 cents).*
2.MD.8b. Fluently use a calendar to answer simple real world problems such as “How many weeks are in a year?” or “James gets a $5 allowance every 2 months, how much money will he have at the end of each year?”	

Represent and interpret data	
2.MD.9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	**A.2.MD.9-10.** Create picture graphs from collected measurement data.
2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph	

Geometry (G)

Reason with shapes and their attributes	
2.G.1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. [8] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	**A.2.G.1.** Identify the common two-dimensional shapes: square, circle, triangle, and rectangle.
2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	Not applicable.
2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	Not applicable. Addressed in **A.4.G.3** and **A.4.NF.1–2**.

[8] Sizes are compared directly or visually, not compared by measuring.

Grade 3

In Grade 3, instruction should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (e.g., fractions with a numerator of one); (3) developing understanding of the structure of rectangular arrays and area; and (4) describing and analyzing two-dimensional shapes. Each critical area is described below.

(1) Students develop an understanding of the meaning of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into three equal parts, the parts are longer than when the ribbon is divided into five equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps—a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication and justify using multiplication to find the area of a rectangle.

(4) Students describe, analyze, and compare the properties of two-dimensional shapes. They compare and classify shapes by their sides and angles and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of a part of a shape as a unit fraction of the whole.

(5) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 3

<table>
<tr><th colspan="2">Operations and Algebraic Thinking (OA)</th></tr>
<tr><td colspan="2">Represent and solve problems involving multiplication and division</td></tr>
<tr><td>3.OA.1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.</td><td rowspan="2">A.3.OA.1-2. Use repeated addition to find the total number of objects and determine the sum.</td></tr>
<tr><td>3.OA.2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.</td></tr>
<tr><td>3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</td><td>Not applicable. Addressed in A.3.OA.1 and A.5.NBT.5.</td></tr>
<tr><td>3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers, with factors 0-10. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 = ?.</td><td>A.3.OA.4. Determine the unknown whole number in an addition or subtraction problem within 20.</td></tr>
<tr><td colspan="2">Understand properties of multiplication and the relationship between multiplication and division</td></tr>
<tr><td>3.OA.5. Apply properties of operations as strategies to multiply and divide.[9] Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)</td><td>Not applicable. Addressed in A.N-CN.2.</td></tr>
</table>

[9] Students need not use formal terms for these properties.

3.OA.6. Understand division as an unknown-factor problem, where a remainder does not exist. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8 with no remainder	Not applicable. Addressed in **A.5.NBT.6–7.**
Multiply and divide within 100	
3.OA.7. ***Fluently*** multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. Know from memory all products of two one-digit numbers; and fully understand the concept when a remainder does not exist under division.	Not applicable. Addressed in **A.7.NS.2.a.** and **A.7.NS.2.b**.
Solve problems involving the four operations, and identify and explain patterns in arithmetic	
3.OA.8. Solve two-step (two operational steps) word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[10] Include problems with whole dollar amounts.	**A.3.OA.8.** Solve one-step *addition or subtraction word problems involving real-life situations* within 20.
3.AO.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.	*Not applicable*
Number and Operations in Base Ten (NBT)	
Use place value understanding and properties of operations to perform multi-digit arithmetic[11]	
3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100.	**A.3.NBT.*1-2.*** *Demonstrate an understanding of place value to the tens place.*
3.NBT.2. Fluently add and subtract (including subtracting across zeros) within 1000 using strategies and algorithms based on place value, properties of	**A.3.NBT.*1*-2.** Demonstrate an understanding of place value to *the* tens *place.*

[10] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.
[11] A range of algorithms may be used.

<table>
<tr><td>operations, and/or the relationship between addition and subtraction. Include problems with whole dollar amounts.</td><td></td></tr>
<tr><td>3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.</td><td>A.3.NBT.3. Using vocalization, sign language, augmentive communication, or assistive technology, count by tens to at least 30 using models such as objects, base-10 blocks, or money.</td></tr>
<tr><td colspan="2">Number and Operations—Fractions[12] (NF)</td></tr>
<tr><td colspan="2">Develop an understanding of fractions as numbers</td></tr>
<tr><td>3.NF.1. Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.</td><td rowspan="3">A.3.NF.1–3. Differentiate a fractional part from a whole.</td></tr>
<tr><td>3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from
c. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.</td></tr>
<tr><td>3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. Recognize that comparisons are valid only when the two fractions refer to the same whole.
b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
c. Express whole numbers as fractions, and</td></tr>
</table>

[12] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.

recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
Measurement and Data (MD)	
Solve problems involving measurement and the estimation of intervals of time, liquid volumes, and masses of objects	
3.M.D.1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	**A.3.MD.1.** *Using vocalization, sign language, augmentive communication, or assistive technology,* tell time to the hour on a digital clock.
3.M.D.2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[13] Add, subtract, multiply, or divide to solve one- step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[14]	**A.3.MD.2.** Identify the appropriate measurement tool *for measuring* mass and volume.
Represent and interpret data	
3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bargraphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	**A.3.MD.3.** Use picture or bar graph*s* to answer questions about data.
3.MD.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where	**A.3.MD.4.** Measure the length of objects *to the nearest whole unit* using standard tools such as rulers, yardsticks, and meter sticks.

[13] Excludes compound units such as cm^3 and finding the geometric volume of a container

[14] Excludes multiplicative comparison problems (e.g., problems involving notions of "times as much")

<table>
<tr><td>the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</td><td></td></tr>
<tr><td colspan="2">Geometric measurement: Understand concepts of area and relate area to multiplication and to addition</td></tr>
<tr><td>3.MD.5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</td><td rowspan="3">Not applicable. Addressed in A.4.MD.2.</td></tr>
<tr><td>3.MD.6. Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units).</td></tr>
<tr><td>3.MD.7. Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole-number side lengths (where factors can be between 1 and 10, inclusively) in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole- number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
d. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. Recognize area as additive.</td></tr>
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<table>
<tr><th colspan="2">Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures</th></tr>
<tr><td>3.MD.8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.</td><td>Not applicable.
Addressed in A.7.G.4. and A.8.G.9.</td></tr>
<tr><th colspan="2">Geometry (G)</th></tr>
<tr><th colspan="2">Reason with shapes and their attributes</th></tr>
<tr><td>3.G.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</td><td>A.3.G.1. Use vocalization, sign language, augmentive communication or assistive technology to describe the attributes of two-dimensional shapes.</td></tr>
<tr><td>3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.</td><td>A.3.G.2. Recognize that shapes can be partitioned into equal areas.</td></tr>
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Grade 4

In Grade 4, instruction should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. Each critical area is described below.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (e.g., equal-sized groups, arrays, and area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers, understand and explain why the procedures work based on place value and properties of operations, and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop an understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of the properties of two-dimensional objects and the use of them to solve problems involving symmetry.

(4) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 4

Operations and Algebraic Thinking (OA)

Represent and solve problems involving multiplication and division	
4.OA.1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	**A.4.OA.1-2.** Demonstrate the connection between repeated addition and multiplication.
4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]	
4.OA.3. Solve multistep (two or more operational steps) word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**A.4.OA.3.** Solve one-step word problems *involving real-life situations* using addition or subtraction within 100 *without regrouping*.
Gain familiarity with factors and multiples	
4.OA.4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	**A.4.OA.4.** Show *how a whole number is a result of two factors.*

Generate and analyze patterns	
4.AO.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	**A.4.OA.5.** Use repeating patterns to make predictions.

Number and Operations in Base Ten[15] (NBT)

Generalize place value understanding for multi-digit whole numbers	
4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	Not applicable. Addressed in **A.5.NBT.1**.
4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	**A.4.NBT.2**. Compare whole numbers to 10 using symbols (*e.g.,* <, >, =).
4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place.	**A.4.NBT.3.** Round any whole number 0-30 to the nearest ten.
Use place value understanding and properties of operations to perform multi-digit arithmetic	
4.NBT.4. ***Fluently*** add and subtract (including subtracting across zeros) multi-digit whole numbers using the standard algorithm.	**A.4.NBT.4.** Add and subtract two-digit whole numbers.
4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Not applicable. Addressed in **A.4.OA.1**.

[15] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

<table>
<tr><td>4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td><td>Not applicable.</td></tr>
<tr><th colspan="2">Number and Operations—Fractions[16] (NF)</th></tr>
<tr><th colspan="2">Extend understanding of fraction equivalence and ordering</th></tr>
<tr><td>4.NF.1. Recognizing that the value of “n” cannot be 0, explain why a fraction a/b is equivalent to a fraction (n × a)/ (n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</td><td rowspan="2">A.4.NF.1–2. Identify models of one half (1/2) and one fourth (1/4).</td></tr>
<tr><td>4.NF.2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.</td></tr>
</table>

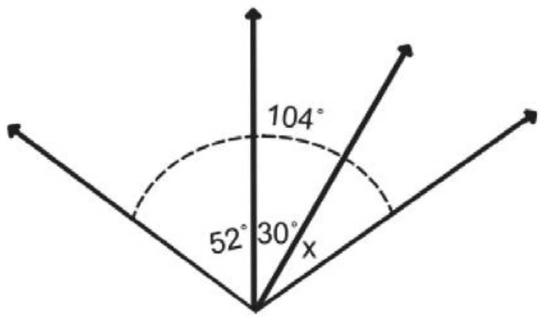
[16] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	
4.NF.3. Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model (including, but not limited to: concrete models, illustrations, tape diagram, number line, area model, etc.). Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	**A.4.NF.3.** Differentiate between whole and half.
4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a/b* as a multiple of *1/b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be	Not applicable. Addressed in **A.4.OA.1–2.** and **A.5.NBT.5**.

needed? Between what two whole numbers do you expect your answer to lie?	
Understand decimal notation for fractions and compare decimal fractions	
4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[17] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*	
4.NF.6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*.	Not applicable. Addressed in **A.7.NS.2.c-d**.
4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	
Measurement and Data (MD)	
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit	
4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm, mm; kg, g, mg; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36)…	**A.4.MD.1.** Identify the smaller measurement unit that comprises a larger unit within a measurement system (*e.g.,* inches/foot, centimeter/meter, minutes/hour).
4.MD.2. Use the four operations to solve word problems involving: • intervals of time, • money, • distances,	**A.4.MD.2.a.** Tell time using a digital clock. Tell time to the nearest hour using an analog clock.
	A.4.MD.2.b. Measure mass or volume using standard tools.

[17] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

• liquid volumes, • masses of objects, including problems *involving simple fractions or decimals*, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	**A.4.MD.2.c.** Use standard measurement to compare lengths of objects. **A.4.MD.2.d.** Identify coins (*e.g.,* penny, nickel, dime, quarter) and their values.
4.MD.3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.	**A.4.MD.3.** Determine the area of a square or rectangle by counting units of measurement (*e.g.,* unit squares).
Represent and interpret data	
4.MD.4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	**A.4.MD.4.a.** Represent data on a picture or bar graph given a model and a graph to complete. **A.4.MD.4.b.** *Using vocalization, sign language, augmentive communication or assistive technology,* interpret *the* data from a picture or bar graph.
Geometric measurement: Understand concepts of angle and measure angles	
4.MD.5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	**A.4.MD.5.** Recognize angles in geometric shapes.
4.MD.6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	**A.4.MD.6.** Identify angles as larger and smaller.

4.MD.7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. Example: Find the missing angle using an equation. 	Not applicable. Addressed in **A.4.G.2.**

Geometry (G)

Draw and identify lines and angles, and classify shapes by properties of their lines and angles

4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	**A.4.G.1.** Recognize parallel lines and intersecting lines.
4.G.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	**A.4.G.2.** *Using vocalization, sign language, augmentive communication or assistive technology,* describe the defining attributes of two-dimensional shapes *(e.g., number of sides, number of angles).*
4.G.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line- symmetric figures and draw lines of symmetry.	**A.4.G.3.** Recognize that lines of symmetry partition shapes into equal areas.

Grade 5

In Grade 5, instruction should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing an understanding of the multiplication of fractions and the division of fractions in limited cases (e.g., unit fractions divided by whole numbers, whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing an understanding of operations with decimals to the hundredths place, and developing fluency with whole number and decimal operations; and (3) developing an understanding of volume. Each critical area is described below.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop an understanding of why division procedures work based on the meaning of base-10 numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understanding of models for decimals, decimal notation, and properties of operations to add and subtract decimals to the hundredths place. They develop fluency in these computations and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (e.g., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to the hundredths place efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure the necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

(4) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 5

Operations and Algebraic Thinking (OA)

Write and interpret numerical expressions

5.OA.1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Not applicable
5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	Not applicable

Analyze patterns and relationships

5.OA.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	**A.5.OA.3.** Identify and extend numerical patterns *(e.g., given the rule "Add 3" and the starting number 0).*

Number and Operations in Base Ten (NBT)

Understand the place value system

5.NBT.1. "*In the number 3.33, the underlined digit represents 3/10, which is 10 times the amount represented by the digit to its right (3/100) and is 1/10 the amount represented by the digit to its left (3)).*	**A.5.NBT.1.** Compare *base-10 models* up to 99 using *symbols (<, >, =).*
5.NBT.2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	**A.5.NBT.2.** Use the number of zeros in numbers that are powers of 10 to determine which values are equal, greater than, or less than.
5.NBT.3. Read, write, and compare decimals to thousandths.	**A.5.NBT.3.** Compare whole numbers up to 100

<table>
<tr><td>a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) +2 × (1/1000).
b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.</td><td>using symbols (<, >, =).</td></tr>
<tr><td>5.NBT.4. Use place value understanding to round decimals to any place.</td><td>A.5.NBT.4. Round two-digit whole numbers to the nearest 10 from 0-90.</td></tr>
<tr><td colspan="2">Perform operations with multi-digit whole numbers and with decimals to the hundredths place</td></tr>
<tr><td>5.NBT.5. <u>Fluently</u> multiply multi-digit whole numbers using the standard algorithm.</td><td>A.5.NBT.5. Multiply whole numbers up to 5 × 5.</td></tr>
<tr><td>5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td><td rowspan="2">A.5.NBT.6–7. Illustrate the concept of division using fair and equal shares.</td></tr>
<tr><td>5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models (to include, but not limited to: base ten blocks, decimal tiles, etc.) or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</td></tr>
<tr><td colspan="2">Number and Operations—Fractions (NF)</td></tr>
<tr><td colspan="2">Use equivalent fractions as a strategy to add and subtract fractions</td></tr>
<tr><td>5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)</td><td>A.5.NF.1. Identify models of halves (e.g., 1/2, 2/2) and fourths (e.g., 1/4, 2/4, 3/4, 4/4).</td></tr>
</table>

5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	**A.5.NF.2.** Identify models of thirds (e.g., 1/3. 2/3, 3/3) and tenths (e.g., 1/10, 2/10, 3/10, 4/10, 5/10, 6/10, 7/10, 8/10, 9/10, 10/10).
Apply and extend previous understandings of multiplication and division to multiply and divide fractions	
5.NF.3. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Not applicable. Addressed in **A.6.RP.1**.
5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (*a/b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* = *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Not applicable.

5.NF.5. Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	Not applicable.
5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	Not applicable. Addressed in **A.10.N-CN.2.b**.
5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[18] a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3*. b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the	Not applicable. Addressed in **A.7.NS.2.b**.

[18] Students able to multiply fractions in general can develop strategies to divide fractions in general by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*	
Measurement and Data (MD)	
Convert like measurement units within a given measurement system	
5.MD.1. Convert among different-sized standard measurement units within a given measurement system (customary and metric) (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	**A.5.MD.1.a.** Tell time using an analog or digital clock to the half or quarter hour.
	A.5.MD.1.b. Use standard units to measure the weight and length of objects.
	A.5.MD.1.c. Indicate the relative value of collections of coins.
Represent and interpret data.	
5.MD.2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	**A.5.MD.2.** Represent and interpret data on a picture, line plot, or bar graph.
Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition	
5.MD.3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	**A.5.MD.3.** Identify common three-dimensional shapes *(e.g., sphere, cylinder, cone).*
5.MD.4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	**A.5.MD.4–5.** Determine the volume of a rectangular prism by counting units of measure*ment* (*e.g.,* unit cubes).
5.MD.5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism	**A.5.MD.4–5.** Determine the volume of a rectangular prism by counting units of measure*ment* (*e.g.,* unit cubes).

with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. Recognize volume as additive. Find volumes of solid figures composed of two non- overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	
Geometry (G)	
Graph points on the coordinate plane to solve real-world and mathematical problems	
5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).	**A.5.G.1-4.** Sort two-dimensional figures and identify the attributes (*e.g.,* angles, number of sides, corners, color) they have in common.
5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	
Classify two-dimensional figures into categories based on their properties	
5.G.3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	**A.5.G.1-4.** Sort two-dimensional figures and identify the attributes (e.g., angles, number of sides, corners, color) they have in common.

5.G.4. Classify two-dimensional figures in a hierarchy based on properties.	

Alternate Academic Achievement Standards for Mathematics (Grades 6-8)

Grade 6

In Grade 6, instruction should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing an understanding of the division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing an understanding of statistical thinking. Each critical area is described below.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending to, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understanding of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations, such as $3x = y$, to describe relationships between quantities.

(4) Building on and reinforcing their understanding of numbers, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (e.g., interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in

which the data were collected.

(5) Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

(6) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student’s individualized education program (IEP).

Grade 6

Ratios and Proportional Relationships (RP)	
Understand ratio concepts and use ratio reasoning to solve problems	
6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	**A.6.RP.1.** Demonstrate a simple ratio relationship.
6.RP.2. Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[19]	Not applicable. Addressed in **A.7.RP.1–3**.
6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	Not applicable. Addressed in **A.8.F.1–3**.

[19] Expectations for unit rates in this grade are limited to non-complex fractions.

The Number System (NS)	
Apply and extend previous understandings of multiplication and division to divide fractions by fractions	
6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	**A.6.NS.1.** Compare the relationships between two unit fractions.
Compute fluently with multi-digit numbers and find common factors and multiples	
6.NS.2. ***Fluently*** divide multi-digit numbers using the standard algorithm.	**A.6.NS.2.** Apply the concept of fair share and equal shares to divide.
6.NS.3. ***Fluently*** add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	**A.6.NS.3.** Solve two-factor multiplication problems with products up to 50 using concrete objects and/or a calculator.
6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*	Not applicable.
Apply and extend previous understandings of numbers to the system of rational numbers	
6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**A.6.NS.5–8.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).
6.NS.6. Understand a rational number as a point on	

the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	
6.NS.7. Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right*. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C >-7 °C to express the fact that –3 °C is warmer than -7°C.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*	**A.6.NS.5–8.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).
6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with	**A.6.NS.5–8.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).

the same first coordinate or the same second coordinate.	
6.NS.9. Apply and extend previous understandings of addition and subtraction to add and subtract integers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*. b. Understand *p + q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of integers by describing real-world contexts. c. Understand subtraction of integers as adding the additive inverse, *p – q = p + (–q)*. Show that the distance between two integers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract integers.	Not applicable.

Expressions and Equations (EE)

Apply and extend previous understandings of arithmetic to algebraic expressions

6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.	**A.6.EE.1–2.** Identify equivalent number sentences.
6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*	**A.6.EE.1–2.** Identify equivalent number sentences.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.	
6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*	**A.6.EE.3.** Apply the properties of addition to identify equivalent numerical expressions.
6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for*.	Not applicable.
Reason about and solve one-variable equations and inequalities	
6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	**A.6.EE.5–8.** Match an equation to a real-world problem in which variables are used to represent numbers.
6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	
6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	
6.EE.8. Write an inequality of the form $x > c$ or $x < c$ to	

represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	
Represent and analyze quantitative relationships between dependent and independent variables	
6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another. • Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. • Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*	Not applicable.
Geometry (G)	
Solve real-world and mathematical problems involving area, surface area, and volume	
6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	**A.6.G.1.** Solve real-world and mathematical problems about area using unit squares.
6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	**A.6.G.2.** Solve real-world and mathematical problems about volume using unit cubes.
6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.	Not applicable.

6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real- world and mathematical problems.	Not applicable.
Statistics and Probability (SP)	
Develop understanding of statistical variability	
6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	**A.6.SP.1–2.** Display data on a graph or table that shows variability in the data.
6.SP.2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	
6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	Not applicable. Addressed in **A.S-ID.4**.
Summarize and describe distributions	
6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	Not applicable. Addressed in **A.6.SP.1–2**.
6.SP.5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	**A.6.SP.5.** *Using vocalization, sign language, augmentive communication, or assistive technology,* summarize data distributions shown in graphs or tables.

Grade 7

In Grade 7, instruction should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples. Each critical area is described below.

(1) Students extend their understanding of ratios and develop an understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed, temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

(4) Students build on their previous work with single-data distributions to compare two- data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

(5) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 7

<table>
<tr><th colspan="2">Ratios and Proportional Relationships (RP)</th></tr>
<tr><td colspan="2">Analyze proportional relationships and use them to solve real-world and mathematical problems</td></tr>
<tr><td>7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.</td><td rowspan="3">A.7.RP.1–3. Use a ratio to model or describe a relationship.</td></tr>
<tr><td>7.RP.2. Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.</td></tr>
<tr><td>7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</td></tr>
<tr><th colspan="2">The Number System (NS)</th></tr>
<tr><td colspan="2">Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers</td></tr>
<tr><td>7.NS.1. Apply and extend previous understandings of</td><td>A.7.NS.1. Add fractions with like denominators</td></tr>
</table>

addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	(*e.g.,* halves, thirds, fourths, tenths) with sums less than or equal to one.
7.NS.2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.	**A.7.NS.2.a.** Solve multiplication problems with products to 100.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then-(*p/q*) = (-*p*)/*q* = *p*/(-*q*). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	**A.7.NS.2.b.** Solve division problems with divisors up to five and also with a divisor of 10 without remainders. **A.7.NS.2.c–d**. Express a fraction with a denominator of 10 as a decimal.
7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers.	**A.7.NS.3.** Compare quantities represented as decimals in real-world examples to tenths.

Expressions and Equations (EE)

Use properties of operations to generate equivalent expressions

7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	**A.7.EE.1.** Use the properties of operations as strategies to demonstrate that expressions are equivalent.
7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*	**A.7.EE.2.** Identify an arithmetic sequence of whole numbers with a whole number common difference.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations	
7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*[20]	Not Applicable
7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms ***<u>fluently</u>***. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? b. Solve word problems leading to inequalities of the form *px + q > r* or *px + q < r*, where *p*, *q*, and *r* are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	**A.7.EE.4.** Use the concept of equality with models to solve one-step addition and subtraction equations.

[20] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Geometry (G)	
Draw, construct, and describe geometrical figures and describe the relationships between them	
7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	**A.7.G.1.** Match two similar geometric shapes that are proportional in size and in the same orientation.
7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	**A.7.G.2.** Recognize geometric shapes with given conditions.
7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	**A.7.G.3.** Match a two-dimensional shape with a three-dimensional shape that shares an attribute.
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume	
7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	**A.7.G.4.** Determine the perimeter of a rectangle by adding the measures of the sides.
7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	**A.7.G.5.** Recognize angles that are acute, obtuse, and right.
7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**A.7.G.6.** Determine the area of a rectangle using the formula for length × width, and confirm the result using tiling or partitioning into unit squares.

Statistics and Probability (SP)	
Use random sampling to draw inferences about a population	
7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	**A.7.SP.1–2.** *Using vocalization, sign language, augmentive communication, or assistive technology,* answer a question related to the collected data from an experiment, given a model of data, or from data collected by the student.
7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	**A.7.SP.1–2.** *Using vocalization, sign language, augmentive communication, or assistive technology,* answer a question related to the collected data from an experiment, given a model of data, or from data collected by the student.
Draw informal comparative inferences about two populations	
7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability on either team; on a dot plot, the separation between the two distributions of heights is noticeable.	**A.7.SP.3.** Compare two sets of data within a single data display such as a picture graph, line plot, or bar graph.
7.SP.4. Use measures of center and measures of variability (i.e. inter-quartile range) for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.	Not applicable. Addressed in **A.S-ID.4**.

Investigate chance processes and develop, use, and evaluate probability models	
7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	**A.7.SP.5–7.** Describe the probability of events occurring as possible or impossible.
7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	
7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	**A.7.SP.5–7.** Describe the probability of events occurring as possible or impossible.

7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	Not applicable.

Grade 8

For Grade 8 math, a <u>one-credit course</u>, instruction should focus on 3 critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence and understanding and applying the Pythagorean Theorem. Each critical area is described below.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade level, fitting the model and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation.

Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on

volume by solving problems involving cones, cylinders, and spheres.

(4) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 8

The Number System (NS)

Know that there are numbers that are not rational, and approximate them by rational numbers

8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	**A.8.NS.1.** Subtract fractions with like denominators (*e.g.,* halves, thirds, fourths, tenths) with minuends less than or equal to one.
8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., rr^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	**A.8.NS.2.a.** Express a fraction with a denominator of 100 as a decimal. **A.8.NS.2.b.** Compare quantities represented as decimals in real-world examples to the hundredths place.

Expressions and Equations (EE)

Work with radicals and integer exponents

8.EE.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	**A.8.EE.1.** Identify the meaning of an exponent (limited to exponents of 2 and 3).
8.EE.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that '12 is irrational.	**A.8.EE.2.** Identify a geometric sequence of whole numbers with a whole number common ratio.
8.EE.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*	**A.8.EE.3–4.** Compose and decompose whole numbers up to 999.
8.EE.4. Perform operations with numbers	**A.8.EE.3–4.** Compose and decompose whole

expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	numbers up to 999.
Understand the connections between proportional relationships, lines, and linear equations	
8.EE.5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	**A.8.EE.5–6.** Graph a simple ratio by connecting the origin to a point representing the ratio in the form of y/x. (*e.g.,* when given a ratio in standard form (2:1), convert to 2/1 and plot the point (1,2).
8.EE.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	
Analyze and solve linear equations and pairs of simultaneous linear equations	
8.EE.7. Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations and inequalities with rational number coefficients, including those whose solutions require expanding expressions using the distributive property and collecting like terms.	**A.8.EE.7.** Solve simple algebraic equations with one variable using addition and subtraction.
8.EE.8. Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.	Not applicable. Addressed in **A.8.EE.5–6**.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	
Functions (F)	
Define, evaluate, and compare functions	
8.F.1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[21]	**A.8.F.1–3.** Given a function table containing at least two complete ordered pairs, identify a missing number that completes another ordered pair (limited to linear functions).
8.F.2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	
8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	
Use functions to model relationships between quantities	
8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and	**A.8.F.4.** Determine the values or rules of a function using a graph or a table.

[21] Function notation is not required in Grade 8.

initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	
8.F.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**A.8.F.5.** Describe how a graph represents a relationship between two quantities.
Geometry (G)	
Understand congruence and similarity using physical models, transparencies, or geometry software	
8.G.1. Verify experimentally the properties of rotations, reflections, and translations a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	**A.8.G.1.** Recognize translations, rotations, and reflections of shapes.
8.G.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	**A.8.G.2.** Identify shapes that are congruent.
8.G.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Not applicable.
8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	**A.8.G.4.** Identify similar shapes with and without rotation.
8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	**A.8.G.5.** Compare any angle to a right angle and describe the angle as greater than, less than, or congruent to a right angle.

Understand and apply the Pythagorean Theorem	
8.G.6. Explain a proof of the Pythagorean Theorem and its converse.	Not applicable.
8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real- world and mathematical problems in two and three dimensions.	Not applicable.
8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	Not applicable.
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres	
8.G.9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	**A.8.G.9.** Use the formulas for perimeter, area, and volume to solve real-world and mathematical problems (limited to perimeter and area of rectangles and volume of rectangular prisms).
Statistics and Probability (SP)	
Investigate patterns of association in bivariate data	
8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Not applicable.
8.SP.2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	Not applicable. Addressed in **A.10.S-ID.1–2.** and **A.10.S-ID.3**.
8.SP.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	Not applicable.
8.SP.4. Understand that patterns of association can also be Addressed in n in bivariate categorical data by displaying frequencies and relative frequencies in	**A.8.SP.4.** Construct a graph or table from given categorical data and compare data categorized in the graph or table.

a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	

Alternate Academic Achievement Standards for Mathematics (Grades 9-12)

High School Overview

The high school standards specify the mathematics that all students should study in order to be college and career ready. The high school standards are listed in conceptual categories:

- Number and quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and probability

Conceptual categories portray a coherent view of high school mathematics. For example, a student's work with functions crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*). The asterisk (*) symbol occasionally appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

High School—Number and Quantity Conceptual Category

Numbers and Number Systems: During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number" (e.g., 1, 2, 3). Soon after that, zero is used to represent "none" and whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand the division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-10 system used to represent whole numbers. During middle school, fractions are augmented by negative fractions to form rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: they have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities: In real-world problems, the answers are usually not numbers but quantities—numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling (e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting average). They also encounter novel situations in which they must conceive the attributes of interest on their own. For example, to find a good measure of overall highway safety they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, for example, when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

High School—Algebra Conceptual Category

Expressions: An expression is a record of computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of an operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and Inequalities: An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers. The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling: Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

High School—Functions Conceptual Category

Functions describe situations where one quantity determines another. For example, the return on a $10,000 investment at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is *T*.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule (e.g., "I'll give you a state, you give me the capital city"); by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates: Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

High School—Modeling Conceptual Category

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

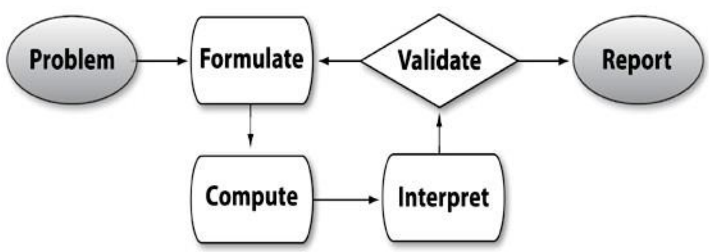
A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people and how it might be distributed
- Planning a table tennis tournament for seven players at a club with four tables where each player plays against every other player
- Designing the layout of the stalls in a school fair in a way to raise as much money as possible
- Analyzing the stopping distance for a car
- Modeling savings account balance, bacterial colony growth, or investment growth
- Engaging in critical path analysis (e.g., applied to the turnaround of an aircraft at an airport)
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism
- Relating population statistics to individual predictions

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes the explosive growth of the exponential function more vivid.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based—for example, the exponential growth of bacterial colonies (until cutoff mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).

High School—Geometry Conceptual Category

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of a material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate that through a point not on a given line there is exactly one parallel line (spherical geometry, in contrast, has no parallel lines).

During high school, students begin to formalize their geometry experiences from elementary and middle school using more precise definitions and developing careful proofs. Later in college, some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. The rigid motions are fundamental—translations, rotations, reflections, and combinations of these—all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—for example, when the reflective symmetry of an isosceles triangle assures that the base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of “same shape” and “scale factor” developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem-solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations: The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

High School—Statistics and Probability Conceptual Category

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take that variability into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell-shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread is not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of comparison might mean depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, the random assignment of individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data is collected are important in drawing conclusions from the data; in critically reviewing the uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model—a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling: Functions may be used to describe data; if the data suggests a linear relationship, then the relationship can be modeled with a regression line and its strength and direction can be expressed through a correlation coefficient.

High School Alternate Math Elements I and II

The fundamental purpose of Alternate Math Elements I and II is to formalize and extend the mathematics that students learned in the middle. The critical areas deepen and extend the understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibits a linear trend. Alternate Math Elements I and II uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final critical area in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The six critical focus areas of this course include:

(1) working with quantities to model and analyze situations;

(2) exploring sequences and their relationships to functions; (3) working and translating between the various forms of linear equations and inequalities; (4) fitting data to a particular model; (5) establishing triangle congruency; and (6) verifying geometric relationships. Each critical area is described below:

   a. By the end of eighth grade, students have had a variety of experiences working with expressions and creating equations. In this first critical focus area, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

   b. In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

   c. By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This critical area builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded in understanding quantities and relationships between them.

d. This critical area builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

e. In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions (e.g., translations, reflections, rotations) and have used these to develop notions about what it means for two objects to be congruent. In this area, students establish triangle congruence criteria based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

f. Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including the properties of special triangles and quadrilaterals and the slopes of parallel and perpendicular lines.

(3) The statements above represent what general education students are expected to master by the end of these courses. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Alternate Math Elements I and II

Numbers and Quantity

The Complex Number System (N-CN)

Perform arithmetic operations with complex numbers

N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers	**A.N.CN.2.a.** *Demonstrate* the commutative, associative, or distributive properties to add, subtract, or multiply whole numbers.
	A.N.CN.2.b. Solve real-world problems involving addition and subtraction of *rational numbers (e.g., whole numbers or* decimals*),* using models when needed.
	A.N.CN.2.c. Solve real-world problems involving *the* multiplication of *rational numbers (e.g.,* whole number *or decimals)*, using models when needed.

Statistics and Probability*

Making Inferences and Justifying Conclusions (S-IC)

Understand and evaluate random processes underlying statistical experiments

S-IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	***A.S-IC.1-2.*** *Select the model that represents the outcome of an event with results from a given data-generated process or demonstration. For example, a model says a spinning coin falls heads up with a probability of 0.5. Would a result of 5 tails in a row cause you to question the model?*
S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.

S-CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).*	**A.S-CP.1-5.** *Given a scenario, select the independent or dependent variable (e.g., If I buy 10 tickets that cost $7.00 each the total cost is $70.00. Which variable is independent?)*
S-CP.2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*	

S-CP.3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.*	**A.S-CP.1-5.** *Given a scenario, select the independent or dependent variable (e.g., If I buy 10 tickets that cost $7.00 each the total cost is $70.00. Which variable is independent?)*
S-CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.**	
S-CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.**	
Geometry	
Congruence (G-CO)	
Experiment with transformations in the plane	
G-CO.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	**A.G-CO.1.** *Demonstrate* perpendicular lines, parallel lines, and line segments; angles; and circles *(e.g., draw, model, identify, create)*
G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	***A.G-CO.2-4.*** *Not Applicable.*
G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	

G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	**A.G-CO.5.** *Identify and model characteristics of a geometric figure that has undergone a transformation (e.g., reflection, rotation, translation).*
Understand congruence in terms of rigid motions	
G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	**A.G-CO.6-8.** *Select* corresponding congruent and similar parts of shapes.
G-CO.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
G-CO.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
Use coordinates to prove simple geometric theorems algebraically	
G-GPE.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*	**A.G-GPE.7.** Find perimeters *or* areas of squares and rectangles to solve real-world problems.
Geometry	
Geometric Measurement and Dimension (G-GMD)	
Explain volume formulas and use them to solve problems	
G-GMD.1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	**A.G-GMD.1-3.** *Compare and contrast* the volume of *various geometric figures.*
G.GMD.2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	
G.GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*	

Visualize relationships between two-dimensional and three-dimensional objects	
G.GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**A.G-GMD.4.** *Given a cross section of a three-dimensional object,* identify the shapes of two-dimensional cross-sections.
Geometry	
Modeling with Geometry (G-MG)	
G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*	**A.G-MG.1-3.** Use geometric shapes to describe real-life objects.
G-MG.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*	
G-MG.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	

Alternate Math Elements III and Alternate Algebra Elements

It is in Alternate Math Elements III and Alternate Algebra Elements that students pull together and apply the accumulation of learning that they have obtained from their previous courses, with content grouped into four critical areas that are organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The four critical areas of this course include (1) working extensively with statistics and probability; (2) culminating work with the Fundamental Theorem of Algebra; (3) understanding periodic phenomena; and (4) exploring function fitting.

Each critical area is described below:

(1) In this area, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role randomness and careful design play in the conclusions that can be drawn.

(2) This area develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-10 computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The area culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except zero. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

(3) Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define zero, one, two, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

(4) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing the appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as “the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions” is at the heart of this area. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

(5) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student’s individualized education program (IEP).

Alternate Math Elements III and Alternate Algebra Elements

Number and Quantity	
The Real Number System (N-RN)	
Extend the properties of exponents to rational exponents	
N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $[5^{1/3}]^3 = 5^{(1/3)3}$ *to hold, so* $[5^{1/3}]^3$ *must equal 5.*	**A.N-RN.1.** Determine the value of a quantity that is squared or cubed.
Quantities (N-Q) *	
Reason quantitatively and use units to solve problems	
N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*	**A.N-Q.1-3.** *Using vocalization, sign language, augmentive communication, or assistive technology,* express quantities to the appropriate precision of measurement.
N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.*	
N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*	
Algebra	
Seeing Structure in Expressions (A-SSE)	
Interpret the structure of expressions	
A-SSE.1. Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*	**A.A-SSE.1.** Identify an algebraic expression involving *addition or subtraction* to represent a real-world problem.
Write expressions in equivalent forms to solve problems	
A-SSE.3. Choose and produce an equivalent form of	**A.A-SSE.3.** Solve simple algebraic equations with

<table>
<tr><td>an expression to reveal and explain properties of the quantity represented by the expression.*
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $[1.15^{1/12}]^{12t}$ "" 1.012^{12t} to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</td><td>one variable using multiplication and division.</td></tr>
<tr><td>A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.*</td><td>A.A-SSE.4. Determine the successive term in a geometric sequence given the common ratio.</td></tr>
<tr><td colspan="2">Creating Equations (A-CED) *</td></tr>
<tr><td colspan="2">Create equations that describe numbers or relationships</td></tr>
<tr><td>A-CED.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*</td><td>A.A-CED.1. Select an equation or inequality involving one operation with one variable that represents a real-world problem.</td></tr>
<tr><td>A-CED.2. Create equations in two variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [Note this standard appears in future courses with a slight variation in the standard language.]</td><td rowspan="3">A.A-CED.2-4. Solve one-step equations or inequalities.</td></tr>
<tr><td>A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*</td></tr>
<tr><td>A-CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.*</td></tr>
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<table>
<tr><th colspan="2">Reasoning with Equations and Inequalities (A-REI)</th></tr>
<tr><th colspan="2">Represent and solve equations and inequalities graphically</th></tr>
<tr><td>A-REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</td><td rowspan="3">A-REI.10-12. Interpret the meaning of a point on the graph of a line. For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost.</td></tr>
<tr><td>A-REI.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, quadratic, absolute value, and exponential functions. *</td></tr>
<tr><td>A-REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</td></tr>
<tr><th colspan="2">Functions</th></tr>
<tr><th colspan="2">Interpreting Functions (F-IF)</th></tr>
<tr><th colspan="2">Understand the concept of a function and use function notation</th></tr>
<tr><td>F-IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).</td><td>A.F-IF.1. Given a table or graph, identify the domain and range values using positive numbers 1-20.</td></tr>
<tr><td>F-IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</td><td>A.F-IF.2. Use the vertical line test to determine if a given relation is a function.</td></tr>
<tr><td>F-IF.3. Recognize that sequences are functions whose domain is a subset of the integers.</td><td>A.F-IF.3. Using vocalization, sign language, augmentive communication, or assistive technology, describe the rule in a simple sequence given the domain and range using positive numbers less than 20.</td></tr>
</table>

<table>
<tr><th colspan="2">Interpret functions that arise in applications in terms of the context</th></tr>
<tr><td>F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*</td><td rowspan="3">A.F-IF.4-6. Given graphs that represent linear functions, interpret different rates of change (e.g., Which is faster or slower?).</td></tr>
<tr><td>F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*</td></tr>
<tr><td>F-IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*</td></tr>
<tr><th colspan="2">Building Functions (F-BF)</th></tr>
<tr><th colspan="2">Build a function that models a relationship between two quantities</th></tr>
<tr><td>F-BF.1. Write a function that describes a relationship between two quantities.*
a. Determine an explicit expression or steps for calculation from a context</td><td>A.F-BF.1. Select the appropriate graphical representation (e.g., first quadrant) given a situation involving a constant rate of change (e.g., slope).</td></tr>
<tr><td>F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*</td><td>A.F-BF.2. Given arithmetic or geometric sequence, identify the graph that models the given rule.</td></tr>
<tr><th colspan="2">Linear, Quadratic, and Exponential Models (F-LE) *</th></tr>
<tr><th colspan="2">Construct and compare linear, quadratic, and exponential models and solve problems</th></tr>
<tr><td>F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.*
a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</td><td>A.F-LE.1-3. Model a simple linear function such as y=mx to show that these functions increase by equal amounts over equal intervals. Given a simple linear function, select the model that represents an increase by equal amounts over equal intervals.</td></tr>
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<table>
<tr><td>F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</td><td rowspan="2">A.F-LE.1-3. Model a simple linear function such as y=mx to show that these functions increase by equal amounts over equal intervals. Given a simple linear function, select the model that represents an increase by equal amounts over equal intervals.</td></tr>
<tr><td>F-LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*</td></tr>
<tr><th colspan="2">Statistics and Probability *</th></tr>
<tr><th colspan="2">Interpreting Categorical and Quantitative Data (S-ID)</th></tr>
<tr><th colspan="2">Summarize, represent, and interpret data on a single count or measurement variable</th></tr>
<tr><td>S-ID.1. Represent and analyze data with plots on the real number line (dot plots, histograms, and box plots).*</td><td rowspan="2">A.S-ID.1-2. Given data, construct a simple graph (e.g., line, pie, bar, picture) or table and interpret the data.</td></tr>
<tr><td>S-ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*</td></tr>
<tr><td>S-ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*</td><td>A.S-ID.3. Interpret general trends on a graph or chart.</td></tr>
<tr><td>S-ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*</td><td>A.S-ID.4. Calculate the mean of a given data set (using whole numbers 1-20).</td></tr>
</table>

MISSISSIPPI DEPARTMENT OF EDUCATION

Ensuring a bright future for every child

2019 Mississippi Alternate Academic Achievement Standards for Mathematics

Effective Date: 2019-2020 School Year

2019 Mississippi Alternate Academic Achievement Standards for Mathematics

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Introduction

The Mississippi Department of Education (MDE) is dedicated to student success, which includes improving student achievement in mathematics and establishing communication skills within a technological environment. The *2019 Mississippi Alternate Academic Achievement Standards* (MS AAAS) provide a consistent, clear understanding of what students are expected to know and be able to do by the end of each grade level or course. The purpose of the alternate standards is to build a bridge from the content in the general education mathematics framework to academic expectations for students with the most significant cognitive disabilities. The standards are designed to be rigorous and relevant to the real world, reflecting the knowledge and skills that students need for success in postsecondary settings.

In special education, prompting is often used to mean a system of structured cues to elicit desired behaviors that otherwise would not occur. In order to clearly communicate that teacher assistance is permitted during instruction of the MS AAAS and is not limited to structured prompting procedures. Guidance and support during instruction should be interpreted as teacher encouragement, general assistance, and informative feedback to support the student.

Purpose

In an effort to closely align instruction for students with significant cognitive disabilities who are progressing toward individualized postsecondary goals, the *MS AAAS for Mathematics* includes grade- and course-specific standards for grades K-12 mathematics. These standards are intended solely for students who have met the criteria for a Significant Cognitive Disability (SCD) as documented in each student's individualized education program (IEP).

This document is designed to provide special education teachers with a basis for curriculum development. As such, this set of alternate standards addresses a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. This framework outlines what knowledge students should obtain and the types of skills students should demonstrate upon completion. The *MS AAAS* are aligned to the Mississippi College- and Career-Readiness Standards (MS CCRS).

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking**; **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

(K-8) Mathematics Domain and (HS) Conceptual Category

Core Idea

Mississippi College- and Career-Readiness Standards (MS CCRS)

Mississippi Alternate Academic Achievement Standards (MS AAAS)

Grade K

Counting and Cardinality (CC)	
Know number names and the count sequence	
K.CC.1. Count to 100 by ones and by tens.	**A.K.CC.1.** Using vocalization, sign language, augmentive communication, or assistive technology, count to 10 by ones starting with one.
K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	Not applicable. Addressed in **A.2.NBT.2.b**.
K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	Not applicable. Addressed in **A.2.NBT.3**.

Implementation

The *2019 MS AAAS for Mathematics* will be implemented beginning in the 2019-2020 school year.

Technology

The MDE strongly encourages the use of technology in all mathematics classrooms. Technology is essential in teaching and learning mathematics; it influences the mathematics taught and enhances student learning. Calculators are often an allowable accommodation. Please consider students' individual learning needs when using technology in the classroom.

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~~Lydia Lankford~~	~~Madison School District~~
~~Cyndi Keene~~	~~Louisville School District~~
~~LaNell Kellum~~	~~Mississippi State University RCU~~
~~Misty McMahon~~	~~Ocean Springs School District~~
~~Judy Middleton~~	~~Mississippi State University RCU~~
~~Roslyn Miller~~	~~Mississippi State University RCU~~
~~Bonnie Owen~~	~~Mississippi State University RCU~~
~~Allison Paige Pigott~~	~~Mississippi Department of Education~~
~~Myra Pannell~~	~~Mississippi State University RCU~~
~~Ashley Privett~~	~~Desoto County School District~~
~~Keri Rankin~~	~~Grenada School District~~
~~Brittany Raybon~~	~~North Pike School District~~
~~Bobby L. Richardson~~	~~Mississippi Department of Education~~
~~Amye Rowan~~	~~Pearl School District~~
~~Denise Sibley~~	~~Mississippi State University RCU~~
~~Connie Smith~~	~~Laurel School District~~
~~Pleshette Smith~~	~~Mississippi Department of Education~~
~~Taye Smith~~	~~Clinton School District~~
~~Eric Sumrall~~	~~Hazlehurst School District~~
~~Elizabeth Testa~~	~~Vicksburg-Warren School District~~
~~Jaicia Thomas~~	~~Lamar County School District~~
~~Stacey Todd~~	~~Ocean Springs School District~~
~~Susan Stampley~~	~~Senatobia School District~~
~~Tiffany Withers~~	~~Rankin County School District~~
~~April Young~~	~~Hattiesburg Public School District~~

The following resources served as a foundation for the development of the *2019 MS AAAS for Mathematics*:

- *Mississippi's College and Career Readiness Standards (MS CCRS) for Mathematics*.
- Dynamic Learning Maps Consortium. (2013). *Dynamic Learning Maps Essential Elements for Mathematics*. Lawrence, KS: University of Kansas.

The Mississippi Alternate Academic Achievement Standards are based on the Dynamic Learning Maps Essential Elements (DLM EE), with additional edits and clarifications to better support the needs of Mississippi teachers and students. Standards language in italicized font indicates Mississippi-specific standards or adjustments to the DLM EE.

Alternate Academic Achievement Standards for Mathematics (Grades K-5)

Fluency/Fluently Defined

Throughout the *2019 MS AAAS for Mathematics* Grades K-5 standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: ***fluently***). With respect to student performance and effective in-class instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend one or more grades earlier in the standards than the grade when fluency is finally expected.

Wherever the word *fluently* appears in an *MS AAAS* content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow—fluent isn't halting, stumbling, or reversing oneself.

A key aspect of fluency in this sense is that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade level to allow all students to meet the standards that call explicitly for fluency.

Grade K

In kindergarten, instruction should focus on two critical areas: (1) representing, relating, and operating on whole numbers—initially with sets of objects; and (2) describing shapes and space. More learning time in kindergarten should be devoted to numbers than to other topics. Each critical area is described below.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5 (kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged but is not required). Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three- dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

(3) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade K

Counting and Cardinality (CC)

Know number names and the count sequence	
K.CC.1. Count to 100 by ones and by tens.	**A.K.CC.1.** *Using vocalization, sign language, augmentive communication, or assistive technology,* count to 10 by ones *starting with one*.
K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	Not applicable. Addressed in **A.2.NBT.2.b**.
K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	Not applicable. Addressed in **A.2.NBT.3**.
Count to tell the number of objects	
K.CC.4. Understand the relationship between numbers and quantities; connect counting to cardinality.	**A.K.CC.4.** Demonstrate one-to-one correspondence, pairing each object with one, and only one, number and each number with one, and only one, object.
K.CC.4.a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.	
K.CC.4.b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.	
K.CC.4.c. Understand that each successive number name refers to a quantity that is one larger.	
K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	**A.K.CC.5.** *Using vocalization, sign language, augmentive communication, or assistive technology,* count out up to three objects from a larger set, pairing each object with one, and only one, number name to tell how many.
Compare numbers	
K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[22]	**A.K.CC.6.** Identify whether the number of objects in one group is more or less than (*e.g.,* when the quantities are clearly different) or equal to the number of objects in another group.
K.CC.7. Compare two numbers between 1 and 10 presented as written numerals.	Not applicable. Addressed in **A.2.NBT.4.**

[22] Include groups with up to 10 objects.

Operations and Algebraic Thinking (OA)

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from

K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings[3], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	**A.K.OA.1.** *Demonstrate an understanding of* addition as "putting together" or subtraction as "taking from" in everyday activities.
K.OA.2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	Not applicable. Addressed in **A.2.NBT.6–7**.
K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).	Not applicable. Addressed in **A.1.NBT.6**.
K.OA.4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	Not applicable. Addressed in **A.1.NBT.2**.
K.OA.5. Fluently add and subtract within 5.	Not applicable. Addressed in **A.3.OA.4**.

Number and Operations in Base Ten (NBT)

Work with numbers 11-19 to gain foundations for place value

K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	Not applicable. Addressed in **A.1.NBT.4.** and **A.1.NBT.6**.

Measurement and Data (MD)

Describe and compare measurable attributes

<table>
<tr><td>K.MD.1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.</td><td rowspan="2">A.K.MD.1-3. Classify objects according to attributes (e.g., big/small, heavy/light, tall/short).</td></tr>
<tr><td>K.MD.2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children, and describe one child as taller/shorter.</td></tr>
</table>

Classify objects and count the number of objects in each category	
K.MD.3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[23]	**A.K.MD.1-3.** Classify objects according to attributes (*e.g.,* big/small, heavy/light, *tall/short*).
Geometry (G)	
Identify and describe shapes (e.g., squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, spheres)	
K.G.1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.	Not applicable. Addressed in **A.1.G.a**.
K.G.2. Correctly name shapes regardless of their orientations or overall size.	**A.K.G.2–3.** Match shapes of the same size and orientation (*e.g.,* circle, square, rectangle, triangle).
K.G.3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	
Analyze, compare, create, and compose shapes	
K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	Not applicable. Addressed in **A.7.G.1**.
K.G.5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	Not applicable.
K.G.6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	Not applicable. Addressed in **A.1.G.3**.

[23] Limit category counts to less than or equal to 10.

Grade 1

In Grade 1, instruction should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes. Each critical area is described below.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use various models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting to two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop an understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[24]

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build an understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

(5) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics

[24] Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 1

Operations and Algebraic Thinking (OA)	
Represent and solve problems involving addition and subtraction	
1.OA.1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**A.1.OA.1.a**. Represent addition and subtraction with*in five using* objects, fingers, mental images, drawings, sounds (e.g., claps), or acting out situations.
	A.1.OA.1.b. Recognize two groups that have the same or equal quantity.
1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**A.1.OA.2.** *Demonstrate* "putting together" *two sets of objects* to solve *the* problem.
Understand and apply properties of operations and the relationship between addition and subtraction	
1.OA.3. Apply properties of operations as strategies to add and subtract.[25] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*	Not applicable. Addressed in **A.6.A.3.** and **A.N-CN.2**.
1.OA.4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*	Not applicable. Addressed in **A.1.NBT.4.** and **A.1.NBT.6**.
Add and subtract within 20	
1.OA.5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	**A.1.OA.5.a.** Use manipulatives or visual representations to indicate the number that results when adding one more.
	A.1.OA.5.b. Apply knowledge of "one less" to subtract one from a number.

[25] Students need not use formal terms for these properties.

<table>
<tr><td>1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).</td><td>Not applicable. Addressed in A.3.OA.4.</td></tr>
<tr><td colspan="2">Work with addition and subtraction equations</td></tr>
<tr><td>1.OA.7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.</td><td>Not applicable. Addressed in A.1.OA.1.b. and A.2.NBT.5.a.</td></tr>
<tr><td>1.OA.8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = D – 3, 6 + 6 = D.</td><td>Not applicable. Addressed in A.3.OA.4.</td></tr>
<tr><td colspan="2"><h2>Number and Operations in Base Ten (NBT)</h2></td></tr>
<tr><td colspan="2">Extend the counting sequence</td></tr>
<tr><td rowspan="2">1.NBT.1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.</td><td>A.1.NBT.1.a. Count by ones to 30.</td></tr>
<tr><td>A.1.NBT.1.b. Count as many as 10 objects and represent the quantity with the corresponding numeral.</td></tr>
<tr><td colspan="2">Understand place value</td></tr>
<tr><td>1.NBT.2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:</td><td rowspan="4">A.1.NBT.2. Create sets of 10.</td></tr>
<tr><td>1.NBT.2.a. 10 can be thought of as a bundle of ten ones—called a “ten.”</td></tr>
<tr><td>1.NBT.2.b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.</td></tr>
<tr><td>1.NBT.2.c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</td></tr>
</table>

1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**A.1.NBT.3.** *Using vocalization, sign language, augmentive communication, or assistive technology,* compare two groups of 10 or fewer items *using appropriate vocabulary (e.g., more, less, equal)* when the number of items in each group is similar.
Use place value understanding and properties of operations to add and subtract	
1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	**A.1.NBT.4.** Compose numbers less than or equal to five in more than one way.
1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	Not applicable. Addressed in **A.1.OA.5.a.** and **A.1.OA.5.b**.
1.NBT.6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**A.1.NBT.6.** Decompose numbers less than or equal to five in more than one way.
Measurement and Data (MD)	
Measure lengths indirectly and by iterating length units	
1.MD.1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.	**1.MD.1–2**. Compare lengths to identify which is longer/shorter, taller/shorter.
1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same- size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	

Tell and write time with respect to a clock and a calendar	
1.MD.3. Tell and write time in hours and half-hours using analog and digital clocks. **1.MD.3.a**. Tell and write time in hours and half-hours using analog and digital clocks.	**A.1.MD.3.a.** Demonstrate an understanding of the terms tomorrow, yesterday, and today.
1.MD.3.b. Identify the days of the week, the number of days in a week, and the number of weeks in each month.	**A.1.MD.3.b.** Demonstrate an understanding of the terms morning, afternoon, day, and night.
	A.1.MD.3.c. Identify activities that come before, next, and after.
	A.1.MD.3.d. Demonstrate an understanding that telling time is the same every day.
Represent and interpret data	
1.MD.4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**A.1.MD.4.** Organize data into categories by sorting.
Work with money	
1.MD.5.a. Identify the value of all U.S. coins (penny, nickel, dime, quarter, half-dollar, and dollar coins). Use appropriate cent and dollar notation (e.g., 25¢, $1).	***A1.MD.5.a-d**. Using vocalization, sign language, augmentive communication, or assistive technology, identify U.S. coins by name (e.g., penny, nickel, dime, quarter).*
1.MD.5.b. Know the comparative values of all U.S. coins (e.g., a dime is of greater value than a nickel).	
1.MD.5.c. Count like U.S. coins up to the equivalent of a dollar.	
1.MD.5.d. Find the equivalent value for all greater value U.S. coins using like value smaller coins (e.g., 5 pennies equal 1 nickel; 10 pennies equal dime, but not 1 nickel and 5 pennies equal 1 dime).	

Geometry (G)	
Reason with shapes and their attributes	
1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	**A.1.G.1.** Identify the basic attributes of objects *(e.g., color, overall size)*.
1.G.2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[26]	**A.1.G.2.** Sort shapes of the same size and orientation (*e.g.,* circle, square, rectangle, triangle).
1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as *two of* or *four of* the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	**A.1.G.3.** Put *two pieces together* to make a shape that relates to the whole (*e.g.,* two semicircles to make a circle, two squares to make a rectangle).

[26] Students do not need to learn formal names such as “right rectangular prism.”

Grade 2

In Grade 2, instruction should focus on four critical areas: (1) extending understanding of base-10 notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. Each critical area is described below.

(1) Students extend their understanding of the base-10 system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1,000) written in base-10 notation, recognizing that the digits in each place represent the amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1,000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-10 notation using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measurement (e.g., centimeter, inch), and they use rulers and other measurement tools with the understanding that linear measurement involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

(5) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 2

Operations and Algebraic Thinking (OA)

Represent and solve problems involving addition and subtraction	
2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Not applicable. Addressed in **A.3.OA.4**.
Add and subtract within 20	
2.OA.2. Fluently add and subtract within 20 using mental strategies[27]. By end of Grade 2, know from memory all sums of two one-digit numbers.	Not applicable. Addressed in **A.2.NBT.6–7.** and **A.3.OA.4**.
Work with equal groups of objects to gain foundations for multiplication	
2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	**A.2.OA.3.** Equally distribute even numbers of objects between two groups.
2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	**A.2.OA.4.** Use *repeated* addition to find the *sum* of objects arranged *in* equal groups up to 10.

Number and Operations in Base Ten (NBT)

Understand place value	
2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: 100 can be thought of as a bundle of ten tens — called a “hundred.” The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	**A.2.NBT.1.** Represent numbers up to 30 with sets of tens and ones, using objects in columns or arrays.

[27] See standard 1.OA.C.6. for a list of mental strategies.

2.NBT.2. Count within 1000; skip-count by 5s starting at any number ending in 5 or 0. Skip-count by 10s and 100s starting at any number.	**A.2.NBT.2.a.** *Using vocalization, sign language, augmentive communication, or assistive technology,* count from 1 to 30 (count with meaning; cardinality).
	A.2.NBT.2.b. *Using vocalization, sign language, augmentive communication, or assistive technology,* name the next number in a sequence between 1 and 10.
2.NBT.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	**A.2.NBT.3.** Identify numerals 1 to 30.
2.NBT.4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	**A.2.NBT.4.** *Using vocalization, sign language, augmentive communication, or assistive technology,* compare sets of objects and numbers using appropriate vocabulary (*e.g.,* more, less, equal).
Use place value understanding and properties of operations to add and subtract	
2.NBT.5. ***<u>Fluently</u>*** add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	**A.2.NBT.5.a.** Identify the meaning of the "+" sign (i.e., combine, plus, add), "–" sign (i.e., separate, subtract, take), and the "=" sign (equal).
	A.2.NBT.5.b. Using concrete examples, compose and decompose numbers up to 10 in more than one way.
2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations.	**A.2.NBT.6-7.** Use objects, representations, and numbers (0–20) to add and subtract.
2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	
2.NBT.8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.	Not applicable.
2.NBT.9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[28]	Not applicable.

[28] Explanations may be supported by drawings or objects.

Measurement and Data (MD)

Measure and estimate lengths in standard units	
2.MD.1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	**A.2.MD.1.** Measure the length of objects using non-standard units.
2.MD.2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	Not applicable.
2.MD.3. Estimate lengths using units of inches, feet, centimeters, and meters.	**A.2.MD.3–4.** Order by length using non-standard units.
2.MD.4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	
Relate addition and subtraction to length	
2.MD.5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	**A.2.MD.5.** Increase or decrease length by adding or subtracting unit(s).
2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.	**A.2.MD.6.** Use a number line to add one more *or one less* unit of length.
Work with time with respect to a clock and a calendar, and work with money	
2.MD.7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	**A.2.MD.7**. Identify on a digital clock the hour that matches a routine activity.
2.MD.8a. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*	**A.2.MD.8.** *Identify the* value *of money (e.g., a penny has a value of 1 cent, a nickel has a value of 5 cents).*
2.MD.8b. Fluently use a calendar to answer simple real world problems such as “How many weeks are in a year?” or “James gets a $5 allowance every 2 months, how much money will he have at the end of each year?”	

Represent and interpret data	
2.MD.9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	**A.2.MD.9-10.** Create picture graphs from collected measurement data.
2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph	
Geometry (G)	
Reason with shapes and their attributes	
2.G.1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[29] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	**A.2.G.1.** Identify the common two-dimensional shapes: square, circle, triangle, and rectangle.
2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	Not applicable.
2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	Not applicable. Addressed in **A.4.G.3** and **A.4.NF.1–2**.

[29] Sizes are compared directly or visually, not compared by measuring.

Grade 3

In Grade 3, instruction should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (e.g., fractions with a numerator of one); (3) developing understanding of the structure of rectangular arrays and area; and (4) describing and analyzing two-dimensional shapes. Each critical area is described below.

(1) Students develop an understanding of the meaning of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into three equal parts, the parts are longer than when the ribbon is divided into five equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps—a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication and justify using multiplication to find the area of a rectangle.

(4) Students describe, analyze, and compare the properties of two-dimensional shapes. They compare and classify shapes by their sides and angles and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of a part of a shape as a unit fraction of the whole.

(5) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 3

Operations and Algebraic Thinking (OA)	
Represent and solve problems involving multiplication and division	
3.OA.1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	**A.3.OA.1-2**. Use repeated addition to find the total number of objects and determine the sum.
3.OA.2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	
3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Not applicable. Addressed in **A.3.OA.1** and **A.5.NBT.5**.
3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers, with factors 0-10. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 = ?.*	**A.3.OA.4.** *Determine the unknown whole number in an addition or subtraction problem* within 20.
Understand properties of multiplication and the relationship between multiplication and division	
3.OA.5. Apply properties of operations as strategies to multiply and divide.[30] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	Not applicable. Addressed in **A.N-CN.2**.

[30] Students need not use formal terms for these properties.

3.OA.6. Understand division as an unknown-factor problem, where a remainder does not exist. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8 with no remainder	Not applicable. Addressed in **A.5.NBT.6–7.**
Multiply and divide within 100	
3.OA.7. ***Fluently*** multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. Know from memory all products of two one-digit numbers; and fully understand the concept when a remainder does not exist under division.	Not applicable. Addressed in **A.7.NS.2.a.** and **A.7.NS.2.b**.
Solve problems involving the four operations, and identify and explain patterns in arithmetic	
3.OA.8. Solve two-step (two operational steps) word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[31] Include problems with whole dollar amounts.	**A.3.OA.8.** Solve one-step *addition or subtraction word problems involving real-life situations* within 20.
3.AO.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.	*Not applicable*
Number and Operations in Base Ten (NBT)	
Use place value understanding and properties of operations to perform multi-digit arithmetic[32]	
3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100.	**A.3.NBT.*1-2.*** *Demonstrate an understanding of place value to the tens place.*
3.NBT.2. Fluently add and subtract (including subtracting across zeros) within 1000 using strategies and algorithms based on place value, properties of	**A.3.NBT.*1*-2.** Demonstrate an understanding of place value to *the* tens *place.*

[31] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.
[32] A range of algorithms may be used.

operations, and/or the relationship between addition and subtraction. Include problems with whole dollar amounts.	
3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	**A.3.NBT.3**. *Using vocalization, sign language, augmentive communication, or assistive technology,* count by tens *to at least 30* using models such as objects, base-10 blocks, or money.
Number and Operations—Fractions[33] (NF)	
Develop an understanding of fractions as numbers	
3.NF.1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b*.	**A.3.NF.1–3.** Differentiate a fractional part from a whole.
3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from c. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.	
3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. Recognize that comparisons are valid only when the two fractions refer to the same whole. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and	

[33] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.

recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
Measurement and Data (MD)	
Solve problems involving measurement and the estimation of intervals of time, liquid volumes, and masses of objects	
3.M.D.1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	**A.3.MD.1.** *Using vocalization, sign language, augmentive communication, or assistive technology,* tell time to the hour on a digital clock.
3.M.D.2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[34] Add, subtract, multiply, or divide to solve one- step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[35]	**A.3.MD.2.** Identify the appropriate measurement tool *for measuring* mass and volume.
Represent and interpret data	
3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bargraphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	**A.3.MD.3.** Use picture or bar graph*s* to answer questions about data.
3.MD.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where	**A.3.MD.4.** Measure the length of objects *to the nearest whole unit* using standard tools such as rulers, yardsticks, and meter sticks.

[34] Excludes compound units such as cm^3 and finding the geometric volume of a container

[35] Excludes multiplicative comparison problems (e.g., problems involving notions of "times as much")

<table>
<tr><td>the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</td><td></td></tr>
<tr><td colspan="2">Geometric measurement: Understand concepts of area and relate area to multiplication and to addition</td></tr>
<tr><td>3.MD.5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</td><td rowspan="3">Not applicable. Addressed in A.4.MD.2.</td></tr>
<tr><td>3.MD.6. Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units).</td></tr>
<tr><td>3.MD.7. Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole-number side lengths (where factors can be between 1 and 10, inclusively) in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole- number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
d. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. Recognize area as additive.</td></tr>
</table>

Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures	
3.MD.8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	Not applicable. Addressed in **A.7.G.4.** and **A.8.G.9**.
Geometry (G)	
Reason with shapes and their attributes	
3.G.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	**A.3.G.1.** *Use vocalization, sign language, augmentive communication or assistive technology to* describe the attributes of two-dimensional shapes.
3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	**A.3.G.2.** Recognize that shapes can be partitioned into equal areas.

Grade 4

In Grade 4, instruction should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. Each critical area is described below.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (e.g., equal-sized groups, arrays, and area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers, understand and explain why the procedures work based on place value and properties of operations, and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop an understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of the properties of two-dimensional objects and the use of them to solve problems involving symmetry.

(4) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 4

<table>
<tr><th colspan="2">Operations and Algebraic Thinking (OA)</th></tr>
<tr><td colspan="2">Represent and solve problems involving multiplication and division</td></tr>
<tr><td>4.OA.1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</td><td rowspan="2">A.4.OA.1-2. Demonstrate the connection between repeated addition and multiplication.</td></tr>
<tr><td>4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]</td></tr>
<tr><td>4.OA.3. Solve multistep (two or more operational steps) word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</td><td>A.4.OA.3. Solve one-step word problems involving real-life situations using addition or subtraction within 100 without regrouping.</td></tr>
<tr><td colspan="2">Gain familiarity with factors and multiples</td></tr>
<tr><td>4.OA.4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</td><td>A.4.OA.4. Show how a whole number is a result of two factors.</td></tr>
</table>

Generate and analyze patterns	
4.AO.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	**A.4.OA.5.** Use repeating patterns to make predictions.

Number and Operations in Base Ten[36] (NBT)

Generalize place value understanding for multi-digit whole numbers	
4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	Not applicable. Addressed in **A.5.NBT.1**.
4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	**A.4.NBT.2**. Compare whole numbers to 10 using symbols (*e.g.,* <, >, =).
4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place.	**A.4.NBT.3.** Round any whole number 0-30 to the nearest ten.
Use place value understanding and properties of operations to perform multi-digit arithmetic	
4.NBT.4. ***Fluently*** add and subtract (including subtracting across zeros) multi-digit whole numbers using the standard algorithm.	**A.4.NBT.4.** Add and subtract two-digit whole numbers.
4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Not applicable. Addressed in **A.4.OA.1**.

[36] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

<table>
<tr><td>4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td><td>Not applicable.</td></tr>
<tr><th colspan="2">Number and Operations—Fractions[37] (NF)</th></tr>
<tr><th colspan="2">Extend understanding of fraction equivalence and ordering</th></tr>
<tr><td>4.NF.1. Recognizing that the value of “n” cannot be 0, explain why a fraction a/b is equivalent to a fraction (n × a)/ (n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</td><td rowspan="2">A.4.NF.1–2. Identify models of one half (1/2) and one fourth (1/4).</td></tr>
<tr><td>4.NF.2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.</td></tr>
</table>

[37] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	
4.NF.3. Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model (including, but not limited to: concrete models, illustrations, tape diagram, number line, area model, etc.). Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	**A.4.NF.3.** Differentiate between whole and half.
4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a/b* as a multiple of *1/b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be	Not applicable. Addressed in **A.4.OA.1–2.** and **A.5.NBT.5**.

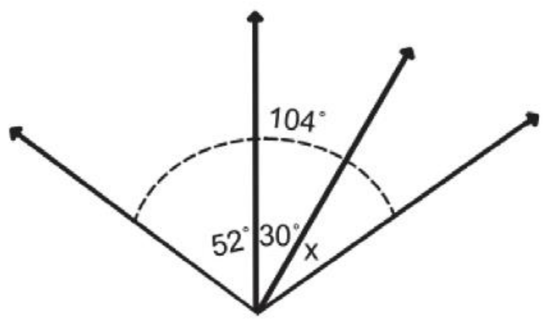
needed? Between what two whole numbers do you expect your answer to lie?	
Understand decimal notation for fractions and compare decimal fractions	
4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[38] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*	Not applicable. Addressed in **A.7.NS.2.c-d**.
4.NF.6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*.	
4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	

Measurement and Data (MD)

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit	
4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm, mm; kg, g, mg; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36)…	**A.4.MD.1.** Identify the smaller measurement unit that comprises a larger unit within a measurement system (*e.g.,* inches/foot, centimeter/meter, minutes/hour).
4.MD.2. Use the four operations to solve word problems involving: • intervals of time, • money, • distances,	**A.4.MD.2.a.** Tell time using a digital clock. Tell time to the nearest hour using an analog clock.
	A.4.MD.2.b. Measure mass or volume using standard tools.

[38] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

• liquid volumes, • masses of objects, including problems *involving simple fractions or decimals*, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	**A.4.MD.2.c.** Use standard measurement to compare lengths of objects.
	A.4.MD.2.d. Identify coins (*e.g.,* penny, nickel, dime, quarter) and their values.
4.MD.3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.	**A.4.MD.3.** Determine the area of a square or rectangle by counting units of measurement (*e.g.,* unit squares).
Represent and interpret data	
4.MD.4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	**A.4.MD.4.a.** Represent data on a picture or bar graph given a model and a graph to complete.
	A.4.MD.4.b. *Using vocalization, sign language, augmentive communication or assistive technology,* interpret *the* data from a picture or bar graph.
Geometric measurement: Understand concepts of angle and measure angles	
4.MD.5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	**A.4.MD.5.** Recognize angles in geometric shapes.
4.MD.6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	**A.4.MD.6.** Identify angles as larger and smaller.

4.MD.7. Recognize angle measure as additive. When an angle is decomposed into non- overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. Example: Find the missing angle using an equation. 	Not applicable. Addressed in **A.4.G.2.**
Geometry (G)	
Draw and identify lines and angles, and classify shapes by properties of their lines and angles	
4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	**A.4.G.1.** Recognize parallel lines and intersecting lines.
4.G.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	**A.4.G.2.** *Using vocalization, sign language, augmentive communication or assistive technology,* describe the defining attributes of two-dimensional shapes *(e.g., number of sides, number of angles).*
4.G.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	**A.4.G.3.** Recognize that lines of symmetry partition shapes into equal areas.

Grade 5

In Grade 5, instruction should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing an understanding of the multiplication of fractions and the division of fractions in limited cases (e.g., unit fractions divided by whole numbers, whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing an understanding of operations with decimals to the hundredths place, and developing fluency with whole number and decimal operations; and (3) developing an understanding of volume. Each critical area is described below.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop an understanding of why division procedures work based on the meaning of base-10 numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understanding of models for decimals, decimal notation, and properties of operations to add and subtract decimals to the hundredths place. They develop fluency in these computations and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (e.g., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to the hundredths place efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure the necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

(4) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student’s individualized education program (IEP).

Grade 5

Operations and Algebraic Thinking (OA)

Write and interpret numerical expressions

5.OA.1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Not applicable
5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	Not applicable

Analyze patterns and relationships

5.OA.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	**A.5.OA.3.** Identify and extend numerical patterns *(e.g., given the rule "Add 3" and the starting number 0).*

Number and Operations in Base Ten (NBT)

Understand the place value system

5.NBT.1. "*In the number 3.33, the underlined digit represents 3/10, which is 10 times the amount represented by the digit to its right (3/100) and is 1/10 the amount represented by the digit to its left (3)).*	**A.5.NBT.1.** Compare *base-10 models* up to 99 using *symbols (<, >, =).*
5.NBT.2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	**A.5.NBT.2.** Use the number of zeros in numbers that are powers of 10 to determine which values are equal, greater than, or less than.
5.NBT.3. Read, write, and compare decimals to thousandths.	**A.5.NBT.3.** Compare whole numbers up to 100

a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) +2 × (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	using symbols (<, >, =).
5.NBT.4. Use place value understanding to round decimals to any place.	**A.5.NBT.4.** Round two-digit whole numbers to the nearest 10 from 0-90.
Perform operations with multi-digit whole numbers and with decimals to the hundredths place	
5.NBT.5. ***Fluently*** multiply multi-digit whole numbers using the standard algorithm.	**A.5.NBT.5.** Multiply whole numbers up to 5 × 5.
5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**A.5.NBT.6–7.** Illustrate the concept of division using fair and equal shares.
5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models (to include, but not limited to: base ten blocks, decimal tiles, etc.) or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	
Number and Operations—Fractions (NF)	
Use equivalent fractions as a strategy to add and subtract fractions	
5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	**A.5.NF.1.** Identify models of halves (*e.g.,* 1/2, 2/2) and fourths (*e.g.,* 1/4, 2/4, 3/4, 4/4).

5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	**A.5.NF.2.** Identify models of thirds (e.g., 1/3. 2/3, 3/3) and tenths (e.g., 1/10, 2/10, 3/10, 4/10, 5/10, 6/10, 7/10, 8/10, 9/10, 10/10).
Apply and extend previous understandings of multiplication and division to multiply and divide fractions	
5.NF.3. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Not applicable. Addressed in **A.6.RP.1**.
5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (*a/b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ⍰⍰⍰*b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Not applicable.

5.NF.5. Interpret multiplication as scaling (resizing), by: c. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. d. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	Not applicable.
5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	Not applicable. Addressed in **A.10.N-CN.2.b**.
5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[39] a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the	Not applicable. Addressed in **A.7.NS.2.b**.

[39] Students able to multiply fractions in general can develop strategies to divide fractions in general by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*	
Measurement and Data (MD)	
Convert like measurement units within a given measurement system	
5.MD.1. Convert among different-sized standard measurement units within a given measurement system (customary and metric) (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	**A.5.MD.1.a.** Tell time using an analog or digital clock to the half or quarter hour. **A.5.MD.1.b.** Use standard units to measure the weight and length of objects. **A.5.MD.1.c.** Indicate the relative value of collections of coins.
Represent and interpret data.	
5.MD.2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	**A.5.MD.2.** Represent and interpret data on a picture, line plot, or bar graph.
Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition	
5.MD.3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	**A.5.MD.3.** Identify common three-dimensional shapes *(e.g., sphere, cylinder, cone).*
5.MD.4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	**A.5.MD.4–5.** Determine the volume of a rectangular prism by counting units of measure*ment* (*e.g.,* unit cubes).

5.MD.5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. Recognize volume as additive. Find volumes of solid figures composed of two non- overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	**A.5.MD.4–5.** Determine the volume of a rectangular prism by counting units of measure*ment* (*e.g.,* unit cubes).
Geometry (G)	
Graph points on the coordinate plane to solve real-world and mathematical problems	
5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).	**A.5.G.1-4.** Sort two-dimensional figures and identify the attributes (*e.g.,* angles, number of sides, corners, color) they have in common.
5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	

<table>
<tr><th colspan="2">Classify two-dimensional figures into categories based on their properties</th></tr>
<tr><td>5.G.3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</td><td rowspan="2">A.5.G.1-4. Sort two-dimensional figures and identify the attributes (e.g., angles, number of sides, corners, color) they have in common.</td></tr>
<tr><td>5.G.4. Classify two-dimensional figures in a hierarchy based on properties.</td></tr>
</table>

Alternate Academic Achievement Standards for Mathematics (Grades 6-8)

Grade 6

In Grade 6, instruction should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing an understanding of the division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing an understanding of statistical thinking. Each critical area is described below.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending to, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understanding of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations, such as $3x = y$, to describe relationships between quantities.

(4) Building on and reinforcing their understanding of numbers, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (e.g., interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in

which the data were collected.

(5) Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

(6) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 6

Ratios and Proportional Relationships (RP)	
Understand ratio concepts and use ratio reasoning to solve problems	
6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	**A.6.RP.1.** Demonstrate a simple ratio relationship.
6.RP.2. Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[40]	Not applicable. Addressed in **A.7.RP.1–3**.
6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	Not applicable. Addressed in **A.8.F.1–3**.

[40] Expectations for unit rates in this grade are limited to non-complex fractions.

The Number System (NS)

Apply and extend previous understandings of multiplication and division to divide fractions by fractions	
6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	**A.6.NS.1.** Compare the relationships between two unit fractions.
Compute fluently with multi-digit numbers and find common factors and multiples	
6.NS.2. ***Fluently*** divide multi-digit numbers using the standard algorithm.	**A.6.NS.2.** Apply the concept of fair share and equal shares to divide.
6.NS.3. ***Fluently*** add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	**A.6.NS.3.** Solve two-factor multiplication problems with products up to 50 using concrete objects and/or a calculator.
6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*	Not applicable.
Apply and extend previous understandings of numbers to the system of rational numbers	
6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**A.6.NS.5–8.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).
6.NS.6. Understand a rational number as a point on	

the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	
6.NS.7. Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right*. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C >-7 °C to express the fact that –3 °C is warmer than -7°C.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*	**A.6.NS.5–8.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).
6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with	**A.6.NS.5–8.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).

the same first coordinate or the same second coordinate.	
6.NS.9. Apply and extend previous understandings of addition and subtraction to add and subtract integers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of integers by describing real-world contexts. c. Understand subtraction of integers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two integers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract integers.	Not applicable.

Expressions and Equations (EE)

Apply and extend previous understandings of arithmetic to algebraic expressions

6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.	**A.6.EE.1–2.** Identify equivalent number sentences.
6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*	**A.6.EE.1–2.** Identify equivalent number sentences.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.	
6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*	**A.6.EE.3.** Apply the properties of addition to identify equivalent numerical expressions.
6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for*.	Not applicable.
Reason about and solve one-variable equations and inequalities	
6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	
6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	**A.6.EE.5–8.** Match an equation to a real-world problem in which variables are used to represent numbers.
6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	
6.EE.8. Write an inequality of the form $x > c$ or $x < c$ to	

represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	
Represent and analyze quantitative relationships between dependent and independent variables	
6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another. • Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. • Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*	Not applicable.

Geometry (G)

Solve real-world and mathematical problems involving area, surface area, and volume	
6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	**A.6.G.1.** Solve real-world and mathematical problems about area using unit squares.
6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	**A.6.G.2.** Solve real-world and mathematical problems about volume using unit cubes.
6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.	Not applicable.

<table>
<tr><td>6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real- world and mathematical problems.</td><td>Not applicable.</td></tr>
<tr><td colspan="2">Statistics and Probability (SP)</td></tr>
<tr><td colspan="2">Develop understanding of statistical variability</td></tr>
<tr><td>6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.</td><td rowspan="2">A.6.SP.1–2. Display data on a graph or table that shows variability in the data.</td></tr>
<tr><td>6.SP.2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.</td></tr>
<tr><td>6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</td><td>Not applicable. Addressed in A.S-ID.4.</td></tr>
<tr><td colspan="2">Summarize and describe distributions</td></tr>
<tr><td>6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.</td><td>Not applicable. Addressed in A.6.SP.1–2.</td></tr>
<tr><td>6.SP.5. Summarize numerical data sets in relation to their context, such as by:
a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.</td><td>A.6.SP.5. Using vocalization, sign language, augmentive communication, or assistive technology, summarize data distributions shown in graphs or tables.</td></tr>
</table>

Grade 7

In Grade 7, instruction should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples. Each critical area is described below.

(1) Students extend their understanding of ratios and develop an understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed, temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

(4) Students build on their previous work with single-data distributions to compare two- data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

(5) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student’s individualized education program (IEP).

Grade 7

<table>
<tr><th colspan="2">Ratios and Proportional Relationships (RP)</th></tr>
<tr><td colspan="2">Analyze proportional relationships and use them to solve real-world and mathematical problems</td></tr>
<tr><td>7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.</td><td rowspan="3">A.7.RP.1–3. Use a ratio to model or describe a relationship.</td></tr>
<tr><td>7.RP.2. Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.</td></tr>
<tr><td>7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</td></tr>
<tr><th colspan="2">The Number System (NS)</th></tr>
<tr><td colspan="2">Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers</td></tr>
<tr><td>7.NS.1. Apply and extend previous understandings of</td><td>A.7.NS.1. Add fractions with like denominators</td></tr>
</table>

addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	(*e.g.,* halves, thirds, fourths, tenths) with sums less than or equal to one.
7.NS.2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.	**A.7.NS.2.a.** Solve multiplication problems with products to 100.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then-(p/q) = ($-p$)/q = p/($-q$). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	**A.7.NS.2.b.** Solve division problems with divisors up to five and also with a divisor of 10 without remainders. **A.7.NS.2.c–d**. Express a fraction with a denominator of 10 as a decimal.
7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers.	**A.7.NS.3.** Compare quantities represented as decimals in real-world examples to tenths.
Expressions and Equations (EE)	
Use properties of operations to generate equivalent expressions	
7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	**A.7.EE.1.** Use the properties of operations as strategies to demonstrate that expressions are equivalent.
7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”*	**A.7.EE.2.** Identify an arithmetic sequence of whole numbers with a whole number common difference.

<table>
<tr><th colspan="2">Solve real-life and mathematical problems using numerical and algebraic expressions and equations</th></tr>
<tr><td>7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.[41]</td><td>Not Applicable</td></tr>
<tr><td>7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms <u>fluently</u>. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.</td><td>A.7.EE.4. Use the concept of equality with models to solve one-step addition and subtraction equations.</td></tr>
</table>

[41] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Geometry (G)	
Draw, construct, and describe geometrical figures and describe the relationships between them	
7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	**A.7.G.1.** Match two similar geometric shapes that are proportional in size and in the same orientation.
7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	**A.7.G.2.** Recognize geometric shapes with given conditions.
7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	**A.7.G.3.** Match a two-dimensional shape with a three-dimensional shape that shares an attribute.
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume	
7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	**A.7.G.4.** Determine the perimeter of a rectangle by adding the measures of the sides.
7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	**A.7.G.5.** Recognize angles that are acute, obtuse, and right.
7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**A.7.G.6.** Determine the area of a rectangle using the formula for length × width, and confirm the result using tiling or partitioning into unit squares.

Statistics and Probability (SP)

Use random sampling to draw inferences about a population	
7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	**A.7.SP.1–2.** *Using vocalization, sign language, augmentive communication, or assistive technology,* answer a question related to the collected data from an experiment, given a model of data, or from data collected by the student.
7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	**A.7.SP.1–2.** *Using vocalization, sign language, augmentive communication, or assistive technology,* answer a question related to the collected data from an experiment, given a model of data, or from data collected by the student.
Draw informal comparative inferences about two populations	
7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability on either team; on a dot plot, the separation between the two distributions of heights is noticeable.	**A.7.SP.3.** Compare two sets of data within a single data display such as a picture graph, line plot, or bar graph.
7.SP.4. Use measures of center and measures of variability (i.e. inter-quartile range) for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.	Not applicable. Addressed in **A.S-ID.4**.

<table>
<tr><th colspan="2">Investigate chance processes and develop, use, and evaluate probability models</th></tr>
<tr><td>7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</td><td rowspan="2">A.7.SP.5–7. Describe the probability of events occurring as possible or impossible.</td></tr>
<tr><td>7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</td></tr>
<tr><td>7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?</td><td>A.7.SP.5–7. Describe the probability of events occurring as possible or impossible.</td></tr>
</table>

7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	Not applicable.

Grade 8

For Grade 8 math, a one-credit course, instruction should focus on 3 critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence and understanding and applying the Pythagorean Theorem. Each critical area is described below.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade level, fitting the model and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation.

Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on

volume by solving problems involving cones, cylinders, and spheres.

(4) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Grade 8

The Number System (NS)

Know that there are numbers that are not rational, and approximate them by rational numbers

8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	**A.8.NS.1.** Subtract fractions with like denominators (*e.g.,* halves, thirds, fourths, tenths) with minuends less than or equal to one.
8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., rr^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	**A.8.NS.2.a.** Express a fraction with a denominator of 100 as a decimal.
	A.8.NS.2.b. Compare quantities represented as decimals in real-world examples to the hundredths place.

Expressions and Equations (EE)

Work with radicals and integer exponents

8.EE.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	**A.8.EE.1.** Identify the meaning of an exponent (limited to exponents of 2 and 3).
8.EE.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that '12 is irrational.	**A.8.EE.2.** Identify a geometric sequence of whole numbers with a whole number common ratio.
8.EE.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*	**A.8.EE.3–4.** Compose and decompose whole numbers up to 999.
8.EE.4. Perform operations with numbers	**A.8.EE.3–4.** Compose and decompose whole

<table>
<tr><td>expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.</td><td>numbers up to 999.</td></tr>
<tr><td colspan="2">Understand the connections between proportional relationships, lines, and linear equations</td></tr>
<tr><td>8.EE.5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</td><td rowspan="2">A.8.EE.5–6. Graph a simple ratio by connecting the origin to a point representing the ratio in the form of y/x. (e.g., when given a ratio in standard form (2:1), convert to 2/1 and plot the point (1,2).</td></tr>
<tr><td>8.EE.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.</td></tr>
<tr><td colspan="2">Analyze and solve linear equations and pairs of simultaneous linear equations</td></tr>
<tr><td>8.EE.7. Solve linear equations in one variable.
a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
b. Solve linear equations and inequalities with rational number coefficients, including those whose solutions require expanding expressions using the distributive property and collecting like terms.</td><td>A.8.EE.7. Solve simple algebraic equations with one variable using addition and subtraction.</td></tr>
<tr><td>8.EE.8. Analyze and solve pairs of simultaneous linear equations.
a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</td><td>Not applicable. Addressed in A.8.EE.5–6.</td></tr>
</table>

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	
Functions (F)	
Define, evaluate, and compare functions	
8.F.1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[42]	**A.8.F.1–3.** Given a function table containing at least two complete ordered pairs, identify a missing number that completes another ordered pair (limited to linear functions).
8.F.2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	
8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	
Use functions to model relationships between quantities	
8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and	**A.8.F.4.** Determine the values or rules of a function using a graph or a table.

[42] Function notation is not required in Grade 8.

initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	
8.F.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**A.8.F.5.** Describe how a graph represents a relationship between two quantities.
Geometry (G)	
Understand congruence and similarity using physical models, transparencies, or geometry software	
8.G.1. Verify experimentally the properties of rotations, reflections, and translations a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	**A.8.G.1.** Recognize translations, rotations, and reflections of shapes.
8.G.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	**A.8.G.2.** Identify shapes that are congruent.
8.G.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Not applicable.
8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	**A.8.G.4.** Identify similar shapes with and without rotation.
8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	**A.8.G.5.** Compare any angle to a right angle and describe the angle as greater than, less than, or congruent to a right angle.

Understand and apply the Pythagorean Theorem	
8.G.6. Explain a proof of the Pythagorean Theorem and its converse.	Not applicable.
8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real- world and mathematical problems in two and three dimensions.	Not applicable.
8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	Not applicable.
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres	
8.G.9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	**A.8.G.9.** Use the formulas for perimeter, area, and volume to solve real-world and mathematical problems (limited to perimeter and area of rectangles and volume of rectangular prisms).
Statistics and Probability (SP)	
Investigate patterns of association in bivariate data	
8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Not applicable.
8.SP.2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	Not applicable. Addressed in **A.10.S-ID.1–2.** and **A.10.S-ID.3**.
8.SP.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	Not applicable.
8.SP.4. Understand that patterns of association can also be Addressed in n in bivariate categorical data by displaying frequencies and relative frequencies in	**A.8.SP.4.** Construct a graph or table from given categorical data and compare data categorized in the graph or table.

a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	

Alternate Academic Achievement Standards for Mathematics (Grades 9-12)

High School Overview

The high school standards specify the mathematics that all students should study in order to be college and career ready. The high school standards are listed in conceptual categories:

- Number and quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and probability

Conceptual categories portray a coherent view of high school mathematics. For example, a student's work with functions crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*). The asterisk (*) symbol occasionally appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

High School—Number and Quantity Conceptual Category

Numbers and Number Systems: During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number" (e.g., 1, 2, 3). Soon after that, zero is used to represent "none" and whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand the division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-10 system used to represent whole numbers. During middle school, fractions are augmented by negative fractions to form rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: they have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities: In real-world problems, the answers are usually not numbers but quantities—numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling (e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting average). They also encounter novel situations in which they must conceive the attributes of interest on their own. For example, to find a good measure of overall highway safety they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, for example, when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

High School—Algebra Conceptual Category

Expressions: An expression is a record of computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of an operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and Inequalities: An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers. The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling: Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

High School—Functions Conceptual Category

Functions describe situations where one quantity determines another. For example, the return on a $10,000 investment at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is *T*.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule (e.g., "I'll give you a state, you give me the capital city"); by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates: Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

High School—Modeling Conceptual Category

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

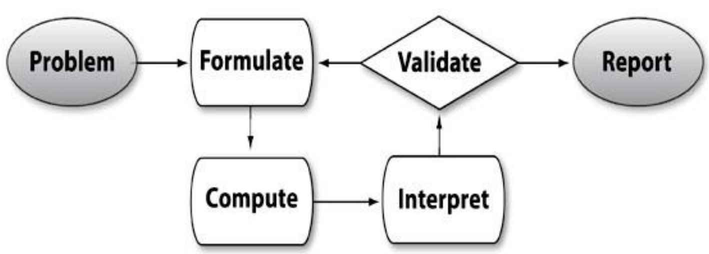
A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people and how it might be distributed
- Planning a table tennis tournament for seven players at a club with four tables where each player plays against every other player
- Designing the layout of the stalls in a school fair in a way to raise as much money as possible
- Analyzing the stopping distance for a car
- Modeling savings account balance, bacterial colony growth, or investment growth
- Engaging in critical path analysis (e.g., applied to the turnaround of an aircraft at an airport)
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism
- Relating population statistics to individual predictions

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes the explosive growth of the exponential function more vivid.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based—for example, the exponential growth of bacterial colonies (until cutoff mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).

High School—Geometry Conceptual Category

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of a material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate that through a point not on a given line there is exactly one parallel line (spherical geometry, in contrast, has no parallel lines).

During high school, students begin to formalize their geometry experiences from elementary and middle school using more precise definitions and developing careful proofs. Later in college, some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. The rigid motions are fundamental—translations, rotations, reflections, and combinations of these—all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—for example, when the reflective symmetry of an isosceles triangle assures that the base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of “same shape” and “scale factor” developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem-solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations: The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

High School—Statistics and Probability Conceptual Category

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take that variability into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell-shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread is not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of comparison might mean depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, the random assignment of individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data is collected are important in drawing conclusions from the data; in critically reviewing the uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model—a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling: Functions may be used to describe data; if the data suggests a linear relationship, then the relationship can be modeled with a regression line and its strength and direction can be expressed through a correlation coefficient.

High School Alternate Math Elements I and II

The fundamental purpose of Alternate Math Elements I and II is to formalize and extend the mathematics that students learned in the middle. The critical areas deepen and extend the understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibits a linear trend. Alternate Math Elements I and II uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final critical area in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The six critical focus areas of this course include:

(1) working with quantities to model and analyze situations;

(2) exploring sequences and their relationships to functions; (3) working and translating between the various forms of linear equations and inequalities; (4) fitting data to a particular model; (5) establishing triangle congruency; and (6) verifying geometric relationships. Each critical area is described below:

   a. By the end of eighth grade, students have had a variety of experiences working with expressions and creating equations. In this first critical focus area, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

   b. In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

   c. By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This critical area builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded in understanding quantities and relationships between them.

d. This critical area builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

e. In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions (e.g., translations, reflections, rotations) and have used these to develop notions about what it means for two objects to be congruent. In this area, students establish triangle congruence criteria based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

f. Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including the properties of special triangles and quadrilaterals and the slopes of parallel and perpendicular lines.

(3) The statements above represent what general education students are expected to master by the end of these courses. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student's individualized education program (IEP).

Alternate Math Elements I and II

Numbers and Quantity

The Complex Number System (N-CN)

Perform arithmetic operations with complex numbers

N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers	**A.N.CN.2.a.** *Demonstrate* the commutative, associative, or distributive properties to add, subtract, or multiply whole numbers.
	A.N.CN.2.b. Solve real-world problems involving addition and subtraction of *rational numbers (e.g., whole numbers or* decimals*),* using models when needed.
	A.N.CN.2.c. Solve real-world problems involving *the* multiplication of *rational numbers (e.g.,* whole number *or decimals*), using models when needed.

Statistics and Probability*

Making Inferences and Justifying Conclusions (S-IC)

Understand and evaluate random processes underlying statistical experiments

S-IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	***A.S-IC.1-2.*** *Select the model that represents the outcome of an event with results from a given data-generated process or demonstration. For example, a model says a spinning coin falls heads up with a probability of 0.5. Would a result of 5 tails in a row cause you to question the model?*
S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.

S-CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).*	**A.S-CP.1-5.** *Given a scenario, select the independent or dependent variable (e.g., If I buy 10 tickets that cost $7.00 each the total cost is $70.00. Which variable is independent?)*
S-CP.2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*	

S-CP.3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.*	
S-CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.**	**A.S-CP.1-5.** *Given a scenario, select the independent or dependent variable (e.g., If I buy 10 tickets that cost $7.00 each the total cost is $70.00. Which variable is independent?)*
S-CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.**	
Geometry	
Congruence (G-CO)	
Experiment with transformations in the plane	
G-CO.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	**A.G-CO.1.** *Demonstrate* perpendicular lines, parallel lines, and line segments; angles; and circles *(e.g., draw, model, identify, create)*
G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	***A.G-CO.2-4.*** *Not Applicable.*
G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	

G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	**A.G-CO.5.** *Identify and model characteristics of a geometric figure that has undergone a transformation (e.g., reflection, rotation, translation).*
Understand congruence in terms of rigid motions	
G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	**A.G-CO.6-8.** *Select* corresponding congruent and similar parts of shapes.
G-CO.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
G-CO.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
Use coordinates to prove simple geometric theorems algebraically	
G-GPE.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*	**A.G-GPE.7.** Find perimeters *or* areas of squares and rectangles to solve real-world problems.
Geometry	
Geometric Measurement and Dimension (G-GMD)	
Explain volume formulas and use them to solve problems	
G-GMD.1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	**A.G-GMD.1-3.** *Compare and contrast* the volume of *various geometric figures.*
G.GMD.2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	
G.GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*	

Visualize relationships between two-dimensional and three-dimensional objects	
G.GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**A.G-GMD.4.** *Given a cross section of a three-dimensional object,* identify the shapes of two-dimensional cross-sections.
Geometry	
Modeling with Geometry (G-MG)	
G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*	**A.G-MG.1-3.** Use geometric shapes to describe real-life objects.
G-MG.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*	
G-MG.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	

Alternate Math Elements III and Alternate Algebra Elements

It is in Alternate Math Elements III and Alternate Algebra Elements that students pull together and apply the accumulation of learning that they have obtained from their previous courses, with content grouped into four critical areas that are organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The four critical areas of this course include (1) working extensively with statistics and probability; (2) culminating work with the Fundamental Theorem of Algebra; (3) understanding periodic phenomena; and (4) exploring function fitting.

Each critical area is described below:

(1) In this area, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role randomness and careful design play in the conclusions that can be drawn.

(2) This area develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-10 computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The area culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except zero. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

(3) Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define zero, one, two, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

(4) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing the appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as “the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions” is at the heart of this area. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

(5) The statements above represent what general education students are expected to master by the end of this grade. The alternate standards address a small number of mathematics standards, representing a breadth, but not depth, of coverage across the entire standards framework. Teaching strategies for students with significant cognitive disabilities should be based on their individual learning goals as outlined in each student’s individualized education program (IEP).

Alternate Math Elements III and Alternate Algebra Elements

<table>
<tr><th colspan="2">Number and Quantity</th></tr>
<tr><th colspan="2">The Real Number System (N-RN)</th></tr>
<tr><th colspan="2">Extend the properties of exponents to rational exponents</th></tr>
<tr><td>N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $[5^{1/3}]^3 = 5^{(1/3)3}$ to hold, so $[5^{1/3}]^3$ must equal 5.</td><td>A.N-RN.1. Determine the value of a quantity that is squared or cubed.</td></tr>
<tr><th colspan="2">Quantities (N-Q) *</th></tr>
<tr><th colspan="2">Reason quantitatively and use units to solve problems</th></tr>
<tr><td>N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*</td><td rowspan="3">A.N-Q.1-3. Using vocalization, sign language, augmentive communication, or assistive technology, express quantities to the appropriate precision of measurement.</td></tr>
<tr><td>N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.*</td></tr>
<tr><td>N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*</td></tr>
<tr><th colspan="2">Algebra</th></tr>
<tr><th colspan="2">Seeing Structure in Expressions (A-SSE)</th></tr>
<tr><th colspan="2">Interpret the structure of expressions</th></tr>
<tr><td>A-SSE.1. Interpret expressions that represent a quantity in terms of its context.*
c. Interpret parts of an expression, such as terms, factors, and coefficients.
d. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.</td><td>A.A-SSE.1. Identify an algebraic expression involving addition or subtraction to represent a real-world problem.</td></tr>
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<tr><th colspan="2">Write expressions in equivalent forms to solve problems</th></tr>
<tr><td>A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^{t} can be rewritten as $[1.15^{1/12}]^{12t}$ "" 1.012^{12t} to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</td><td>A.A-SSE.3. Solve simple algebraic equations with one variable using multiplication and division.</td></tr>
<tr><td>A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.*</td><td>A.A-SSE.4. Determine the successive term in a geometric sequence given the common ratio.</td></tr>
<tr><th colspan="2">Creating Equations (A-CED) *</th></tr>
<tr><th colspan="2">Create equations that describe numbers or relationships</th></tr>
<tr><td>A-CED.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*</td><td>A.A-CED.1. Select an equation or inequality involving one operation with one variable that represents a real-world problem.</td></tr>
<tr><td>A-CED.2. Create equations in two variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [Note this standard appears in future courses with a slight variation in the standard language.]</td><td rowspan="3">A.A-CED.2-4. Solve one-step equations or inequalities.</td></tr>
<tr><td>A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*</td></tr>
<tr><td>A-CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.*</td></tr>
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<tr><th colspan="2">Reasoning with Equations and Inequalities (A-REI)</th></tr>
<tr><th colspan="2">Represent and solve equations and inequalities graphically</th></tr>
<tr><td>A-REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</td><td rowspan="3">A-REI.10-12. Interpret the meaning of a point on the graph of a line. For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost.</td></tr>
<tr><td>A-REI.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, quadratic, absolute value, and exponential functions. *</td></tr>
<tr><td>A-REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</td></tr>
<tr><th colspan="2">Functions</th></tr>
<tr><th colspan="2">Interpreting Functions (F-IF)</th></tr>
<tr><th colspan="2">Understand the concept of a function and use function notation</th></tr>
<tr><td>F-IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).</td><td>A.F-IF.1. Given a table or graph, identify the domain and range values using positive numbers 1-20.</td></tr>
<tr><td>F-IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</td><td>A.F-IF.2. Use the vertical line test to determine if a given relation is a function.</td></tr>
<tr><td>F-IF.3. Recognize that sequences are functions whose domain is a subset of the integers.</td><td>A.F-IF.3. Using vocalization, sign language, augmentive communication, or assistive technology, describe the rule in a simple sequence given the domain and range using positive numbers less than 20.</td></tr>
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<tr><th colspan="2">Interpret functions that arise in applications in terms of the context</th></tr>
<tr><td>F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*</td><td rowspan="3">A.F-IF.4-6. Given graphs that represent linear functions, interpret different rates of change (e.g., Which is faster or slower?).</td></tr>
<tr><td>F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*</td></tr>
<tr><td>F-IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*</td></tr>
<tr><th colspan="2">Building Functions (F-BF)</th></tr>
<tr><th colspan="2">Build a function that models a relationship between two quantities</th></tr>
<tr><td>F-BF.1. Write a function that describes a relationship between two quantities.*
a. Determine an explicit expression or steps for calculation from a context</td><td>A.F-BF.1. Select the appropriate graphical representation (e.g., first quadrant) given a situation involving a constant rate of change (e.g., slope).</td></tr>
<tr><td>F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*</td><td>A.F-BF.2. Given arithmetic or geometric sequence, identify the graph that models the given rule.</td></tr>
<tr><th colspan="2">Linear, Quadratic, and Exponential Models (F-LE) *</th></tr>
<tr><th colspan="2">Construct and compare linear, quadratic, and exponential models and solve problems</th></tr>
<tr><td>F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.*
a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity</td><td>A.F-LE.1-3. Model a simple linear function such as y=mx to show that these functions increase by equal amounts over equal intervals. Given a simple linear function, select the model that represents an increase by equal amounts over equal intervals.</td></tr>
</table>

grows or decays by a constant percent rate per unit interval relative to another.	
F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	**A.F-LE.1-3.** Model a simple linear function such as *y=mx* to show that these functions increase by equal amounts over equal intervals. *Given a simple linear function, select the model that represents an increase by equal amounts over equal intervals.*
F-LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*	

Statistics and Probability *

Interpreting Categorical and Quantitative Data (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable

S-ID.1. Represent and analyze data with plots on the real number line (dot plots, histograms, and box plots).*	**A.S-ID.1-2.** Given data, construct a simple graph (*e.g.,* line, pie, bar, picture) or table and interpret the data.
S-ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*	
S-ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*	**A.S-ID.3.** Interpret general trends on a graph or chart.
S-ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*	**A.S-ID.4.** Calculate the mean of a given data set *(using whole numbers 1-20)*.